Economics of Information

Problem Set 3

1. Consider the following insurance model with adverse selection. The insuree can have a low probability of accident $\underline{\theta}$ or a high one $\overline{\theta} > \underline{\theta}$ with probabilities $\underline{p}$ and $\overline{p}$ respectively. The insuree knows his probability of accident but the insurance company (which is a monopolist and offers a menu of contracts) does not. The insuree has objective function

$$u_1(W_1, W_2, \theta) = (1 - \theta)U(W_1) + \theta U(W_2),$$

where W_1 and W_2 are his net incomes in states of nature 1 (no accident) and 2 (accident) and U is his von Neumann-Morgenstern utility function ($U' > 0.U'' < 0$). With W_0 denoting the insuree's initial wealth and D the monetary damage in case of accident, the risk-neutral insurer's expected utility is

$$u_0(W_1, W_2, \theta) = (1 - \theta)(W_0 - W_1) + \theta(W_0 - D - W_2).$$

Write the individual rationality and incentive compatibility constraints. Compute the optimal menu of contracts chosen by the monopolist, by guessing which of the individual rationality and incentive constraints must be binding.

2. A monopolist faces a single consumer. The consumer has utility $u_1 = \theta q - t$ where q is consumption and t the transfer to the monopolist. The monopolist has cost $\frac{cq^2}{2}$ and offers a sales contract to the consumer. The consumer has reservation utility 0.

   (a) Compute the transfer and the consumption under full information about θ.

   (b) Suppose form now on that the monopolist has incomplete information about θ, which takes the value $\underline{\theta}$ with probability $\underline{p}$ and $\overline{\theta}$ with probability $\overline{p}$. Assume $\underline{\theta} > \overline{p}\overline{\theta}$. The monopolists's utility is

$$\underline{p}(\underline{t} - c\frac{\underline{q}^2}{2}) + \overline{p}(\overline{t} - \frac{c\overline{q}^2}{2}).$$

   Compute the optimal nonlinear tariff. Show that the equilibrium utility of type $\overline{\theta}$ is $\overline{S} = (\overline{\theta} - \underline{\theta})\frac{\underline{\theta} - \overline{p}\overline{\theta}}{c\underline{p}}$.

(c) Suppose now that the consumer can purchase at a fixed cost f a bypass technology that allows him to produce any amount q of the same good at cost $\frac{\tilde{c}q^2}{2}$. Suppose for simplicity that the consumer can only consume the monopolists's good or the alternative good but not both and that

$$\frac{\overline{\theta}^2}{2\tilde{c}} - f > \overline{S} > 0 > \frac{\underline{\theta}^2}{2\tilde{c}} - f.$$

Is the tariff derived earlier still optimal for the monopolist? Discuss what may be optimal for the monopolist – in particular why it may be optimal to have $c\overline{q} > \overline{\theta}$.