Economics of Information

Problem Set 2

1. Consider a first price all pay-auction where both the winner and the loser pay their bids.

    (a) Write down the payoffs of the two players as a function of their value and the two bids

    (b) Show that the equilibrium bid is increasing in the players' value

    (c) Derive the differential equation characterizing the symmetric equilibrium bidding function when values are independently distributed according to the distribution $P(\cdot)$.

    (d) Compute the equilibrium bidding function when the distribution of values is uniform over $[0, 1]$. Compare this bidding function with that obtained under a classical first-price and second-price auction.

    (e) Compute the expected surplus of the seller and compare it with the expected surplus under a classical first-price and second-price auction. What do you observe?

2. The present discounted value of a public good is 1 for all players $i = 1, 2, .., I$. Time is continuous and the rate of interest is r. Each player's cost c of supplying the public good is distributed according to the cumulative distribution function P on $[0, 1]$. Players' types are independent. The public good is supplied if at least one agent supplies it. The good is supplied at the first time at which at least one player chooses to contribute. Thus the game is a kind of war of attrition. Look for a symmetric pure strategy equilibrium using the following outline

    (a) Write down the payoffs of the players according to the times chosen $s_1, .., s_I$ and their cost,

    (b) Argue that the time at which i provides the public good $s_i(c)$ is increasing in c

    (c) Derive the differential equation characterizing the symmetric equilibrium.

    (d) Show that a player's waiting time to supply the good when there are $I - 1$ other players is $I - 1$ times his waiting time when there are two players.

    (e) Show that each player's utility function is increasing in I.

3. This exercise analyzes the first and second price auctions with risk averse bidders and two types per bidder. A bidder with valuation θ has utility $u(\theta - t)$ if he wins and pays transfer t and utility $u(-t)$ if he loses and pays transfer t where u is increasing and concave. The bidders' valuations are independently drawn from the two-type distribution $\underline{\theta}$ with probability $\underline{p}$ and $\overline{\theta}$ with probability $\overline{p}$.

   (a) Show that in the second price auction each player bids his true valuation and so the seller's expected revenue is the same as with risk-neutral bidders.

   (b) Now consider the first-price auction. Compute the mixed strategy distribution of bids for high types $\overline{\theta}$ and show that it first order stochastically dominates the distribution of bids for risk-neutral players. Conclude that under risk aversion the seller prefers a first-price auction to a second-price auction.