Lecture IX: Moral hazard

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The principal agent problem

- From one to two agents: we now consider interactions between two agents: a *principal* and an *agent*
- The principal has the bargaining power and designs a *contract*
- The agent reacts to the principal and chooses an *action* or a *message*
- The principal is risk neutral, the agent is risk averse.

Examples of principal agent relations

- Shareholder (P) Manager (A)
- Landlord (P) Farmer (A)
- Employer (P) Employee (A)
- Insurer (P) Insuree (A)

Asymmetric information

- The agent knows something that the principal cannot observe.
  - *Hidden action* P cannot observe A's action (Moral hazard)
  - *Hidden information* P cannot observe A's type (Adverse selection)
- Because of asymmetric information, the agent can extract a *rent*
- Question: How can one minimize the agent's rent? What is the optimal contract chosen by the principal?

Basic model

- There is a result $x \in X$ (e.g. sales, harvest...). The result is always observed by both parties.
- The agent chooses an effort level e
- The relation between effort and result is not deterministic but stochastic (if deterministic, the principal could deduce the effort from the result)
- Results are ordered $x_1 < x_2 < ..., x_n$
- $\Pr[x = x_i] = p_i(e)$. Suppose that $p_i(e) > 0$ for all i, e

Utilities

- The principal obtains the result x and pays a wage w
- Her utility is given by $B(x - w)$ with $B' > 0$ and $B'' < 0$. *We often assume $B'' = 0$: the principal is risk neutral*
- The agent's utility is separable in wage and effort:

$$u(w, e) = u(w) - v(e),$$

  where $u' > 0, u'' < 0$, $v' > 0, v'' > 0$.
- Conflict of interest: the principal cares about result but not effort, the agent cares about effort but not result

Reservation utilities, participation and contract

- The time sequence of the model is:
  1. P offers a contract, specifying a wage for the agent as a function of observed variables
  2. A accepts or rejects the contract. If rejects, receives a *reservation utility* $\overline{U}$
  3. A chooses effort
  4. Results are realized, wage is paid to A

Contracts with symmetric information

- Suppose that information is symmetric: P observes the effort of A.
- A contract specifies both a payment scheme $w(x_i)$ and an effort level e in order to maximize

$$\sum_i p_i(e)B(x_i - w(x_i)),$$

subject to the constraint

$$\sum p_i(e)u(w(x_i)) - v(e) \geq \overline{U}.$$

The constraint is called the *participation constraint* of the agent.

Optimal risk sharing

- Lagrangian is

$$\mathcal{L} = \sum_i p_i(e)B(x_i - w(x_i)) + \lambda[\overline{U} - (\sum p_i(e)u(w(x_i)) - v(e)),$$

- Fix e. For any x_i, the wage $w(x_i)$ is chosen to maximize the Lagrangian:

$$\frac{\partial \mathcal{L}}{\partial w(x_i)} = -p_i(e)B'(x_i - w(x_i)) + \lambda p_i(e)u'(w(x_i)) = 0.$$

- Hence, for any x_i,

$$\frac{B'(x_i - w(x_i))}{u'(w(x_i)} = \lambda.$$

- *Optimal risk sharing: marginal utilities are equalized for the principal and the agent*

Risk neutral principal, agent

- If P is risk neutral, B' is a constant, so $U'(w(x_i))$ is a constant, ie $w(x_1) = w(x_2) = ...w(x_n)$
- The agent receives a *fixed wage*
- If A is risk neutral, u' is constant so $B'(x_i - w(x_i))$ is constant ie $x_i - w(x_i)$ is a constant
- The agent receives a wage $x_i - k$. This is equivalent to a *fixed franchise* paid to the principal
- Of both P and A are risk averse, the optimal contract is complicated. If A and P have constant absolute risk aversion, the optimal contract is linear

$$w(x_i) = a + bx_i.$$

Optimal choice of effort

- Suppose that P is risk neutral. Let w be the fixed wage.
- For any e, the wage is given by the participation constraint,

$$w = u^{-1}(\overline{U} + v(e)).$$

- and the objective of the principal

$$\sum_i p_i(e)x_i - u^{-1}(\overline{U} + v(e)),$$

- This function is not necessarily concave in e
- If it is concave in e, the solution is given by

$$\sum_i p_i'(e) - u^{-1'}(\overline{U} + v(e))v'(e).$$

Moral hazard

- Moral hard problem arises when the effort of the agent is not observed
- Example: farmer's efforts are not observed but the harvest is
- Salesman's effort is not observed but sales are
- Behavior of driver is not observed but accidents are.

Incentive constraint

- In addition to the participation constraint, there is an *incentive constraint*
- If P wants to induce effort level e^* it must be that

$$\sum p_i(e^*)u(W(x_i)) - v(e^*) \geq \sum p_i(e)u(W(x_i)) - v(e) \forall e,$$

- If there are only two effort levels, the incentive constraint is simple to write
- If there are many effort levels, the condition becomes difficult to write!

Two effort levels, two results

- Suppose that there are two effort levels, e^H and e^L with costs $v(e^H) = c^H > c^L = v(e^L)$
- There are two possible results $x_1 < x_2$ and the probabilities are given by

	x^H	x^L
e^H	p^H	p^L
e^L	q^H	q^L

- with $p^H > q^H$, $p^H + p^L = 1$, $q^H + q^L = 1$

Participation and incentive constraints

- Suppose that P wants to implement the high effort level.
- The participation constraint is:

$$p^H u(w_H) + p^L u(w^L) - c^H \geq \overline{U},$$

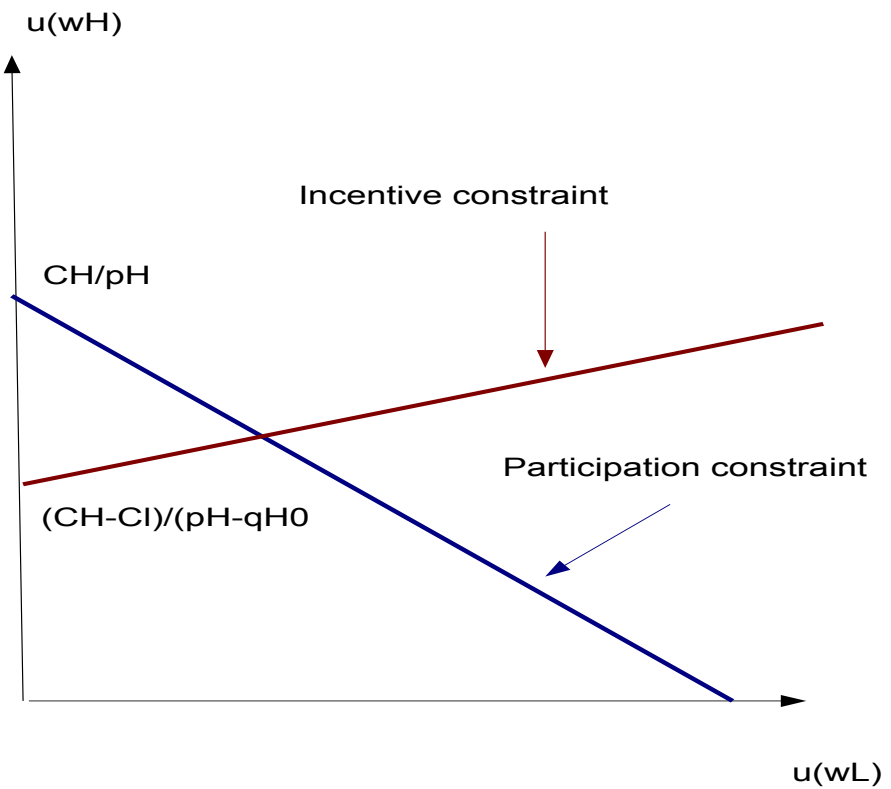
- The incentive constraint is

$$p^H u(w_H) + p^L u(w^L) - c^H \geq q^H u(w_H) + q^L u(w_L) - c^L.$$

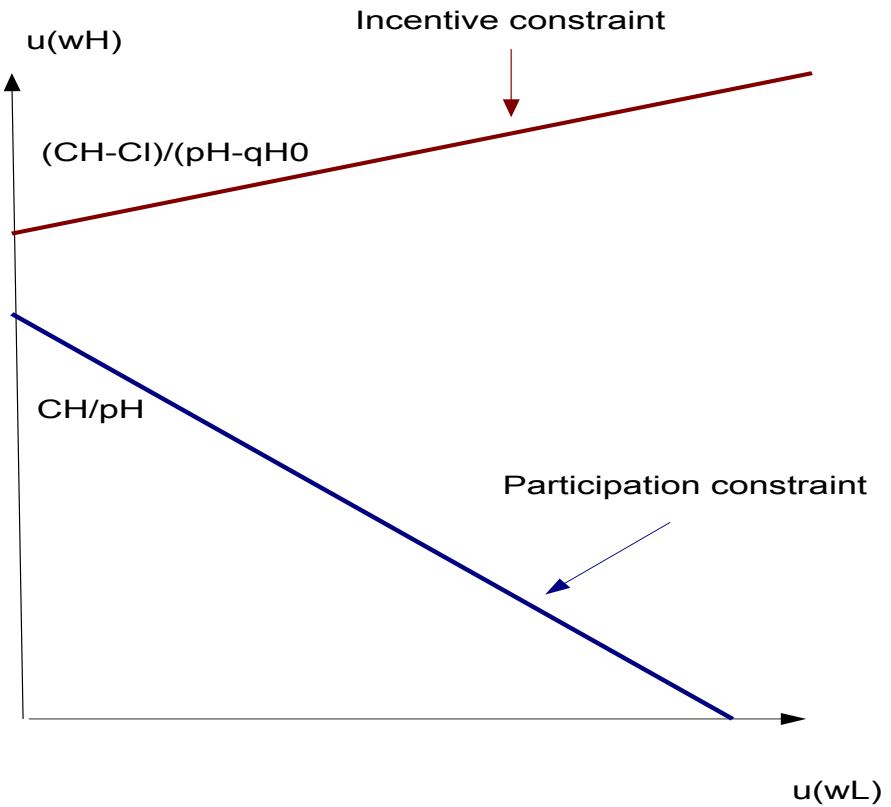
- or

$$(p^H - q^H)u(w^H) + (p^L - q^L)u(w^L) \geq (c^H - c^L).$$

Participation and incentive constraints



Participation and incentive constraints



Principal's profit

- The principal maximizes her profit:

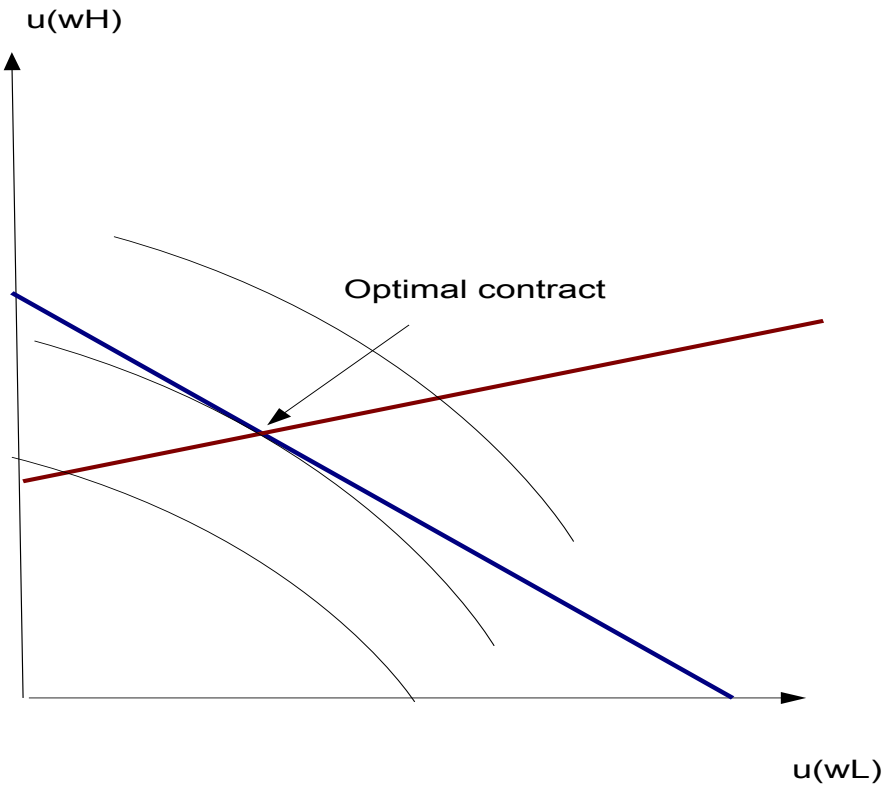
$$p^H(x^H - w^H) + p^L(x^L - w^L).$$

- We consider the isoprofit curves:

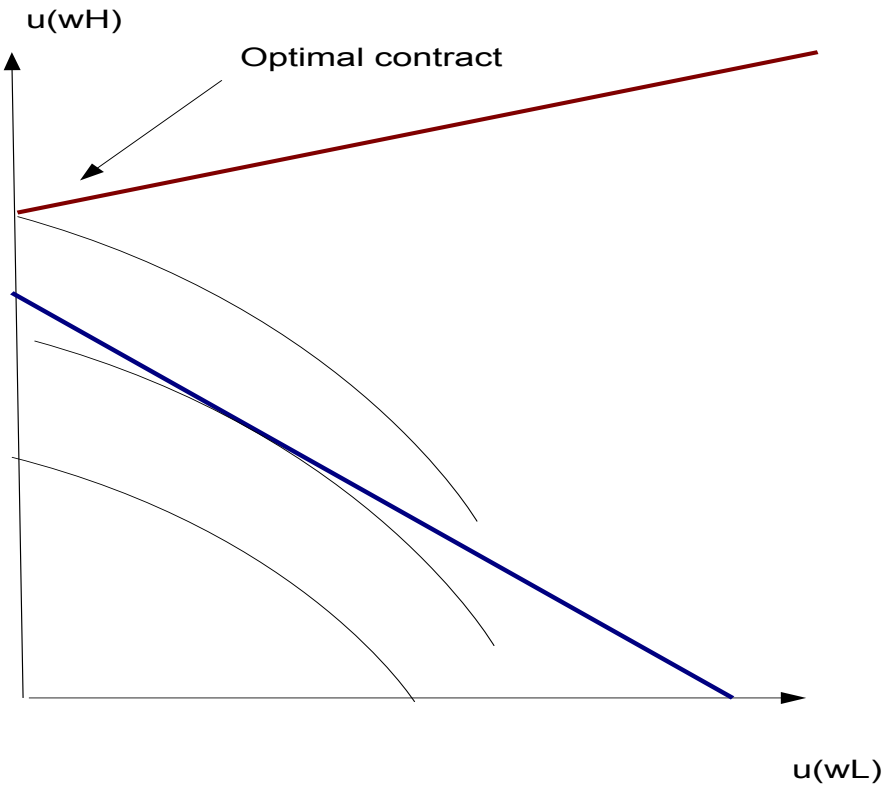
$$p^H(x^H - w^H) + p^L(x^L - w^L) = \pi.$$

- These curves are decreasing curves in the space $(u(w^H), u(w^L)$ and they increase towards the southwest.

Optimal contract



Optimal contract



Interpretation

- The optimal contract always lies on the incentive constraint
- The agent is sometimes pushed down to his reservation utility, sometimes not.

Mathematical solution

- The Lagrangian is

$$\begin{aligned}\mathcal{L} &= p^H(x^H - w^H) + p^L(x^L - w^L) \\ &+ \lambda(\overline{U} - (p^H u(w_H) + p^L u(w^L) - c^H) \\ &+ \mu((c^H - c^L) - (p^H - q^H)u(w^H) + (p^L - q^L)u(w^L))\end{aligned}$$

- The solution is

$$\begin{aligned}1 - \lambda u'(w^H) - \mu(1 - \frac{q^H}{p^H})u'(w^H) &= 0, \\ 1 - \lambda u'(w^L) - \mu(1 - \frac{q^L}{p^L})u'(w^L) &= 0,\end{aligned}$$

Interpretation of the solution

- As $\mu \neq 0$, the optimal wages *are not constant*, $w^H \neq w^L$
- In fact, as $\frac{q^H}{p^H} \leq \frac{q^L}{p^L}$, $w^H > w^L$.

An example

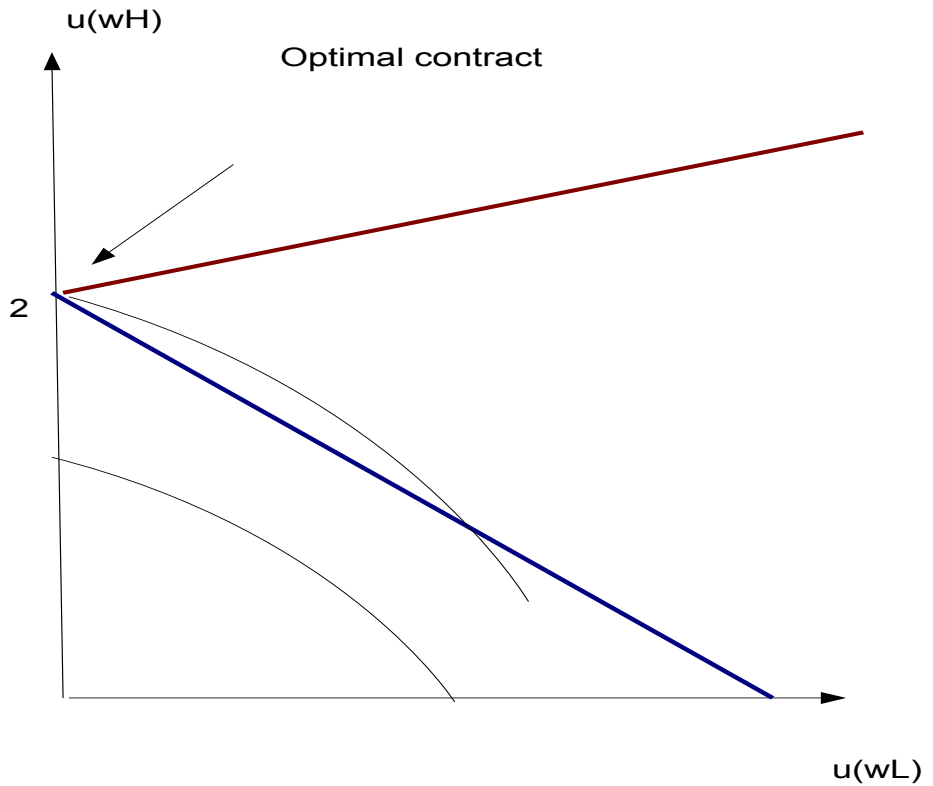
- Suppose $x^L = 0$, $x^H = 20$, $p^H = \frac{1}{2}$, $q^H = \frac{1}{4}$, $c^L = 0$, $c^H = \frac{1}{2}$
- Suppose $u(w) = \sqrt{w}$, $\overline{U} = \frac{1}{2}$
- Participation constraint:

$$\sqrt{w_H} + \sqrt{w_L} \geq 2,$$

- Incentive constraint:

$$sqrtw_H = \sqrt{w_L} + 2.$$

Optimal contract



Optimal contract

- In the optimal contract, $w^H = 4$, $w^L = 0$.
- The profit of P is $\frac{1}{2}(20 - 4) = 8$
- If instead P asks A to make effort e^L, the participation constraint gives

$$\frac{1}{4}\sqrt{w_H} + \frac{3}{4}\sqrt{w_L} = \frac{1}{2}.$$

- As $w_H = w_L$, the wage is $w = \frac{1}{4}$, and the expected profit

$$\frac{20}{4} - \frac{1}{4} = \frac{19}{4} < 8,$$

- so that the optimal effort choice is e^H.

Continuous effort choices

- If the effort choice is continuous, the incentive constraint is replaced by the first order condition, stating that the optimal effort choice of the agent is e^*:

$$\sum_i p_i'(e^*)u(w(x_i)) - v'(e^*) = 0.$$

- This approach is valid only when the agent's profit is concave in e – which is not always the case.
- Replacing in the Lagrangian, we find the solution:

$$\frac{1}{u'(w(x_i))} = \lambda + \mu \frac{p_i'(e)}{p_i(e)}.$$

Other models of moral hazard

- Severity of punishments
- Giving bargaining power to the agent
- Multiple agents: yardstick competition and using information on relative performance
- Multiple principals: competition between principals and common agency.

Summary of Lecture IX

- Consider a contract between two individuals: an agent, a principal
- The principal proposes the contract, the agent accepts or rejects , the contract is executed
- The agent either has hidden information (adverse selection) or his effort is not observable (hidden action)
- In a symmetric information contract, the principal only faces a participation constraint, and risk is optimally shared between the principal and agent
- if the principal is risk neutral, the optimal contract is a fixed wage ; if the agent is risk neutral, a franchise.

Summary of Lecture IX

- When the effort of the agent is not observed, the principal offers an incentive contract, where wages depend on the results.
- The principal faces both a participation and an incentive constraint
- In the optimal contract, the incentive constraint is satisfied with equality ; the agent may or may not be driven to his reservation utility.
- In the optimal contract, higher results are rewarded with higher wages.