Lecture VIII: Cost Minimization and Producer Supply

Francis Bloch and Fabrice Le Lec[1]

[1] Université Paris I

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Cost Minimization vs Profit Maximization

- In the previous lecture, we looked directly at the choice of a profit maximizing firm
- If we want to understand the producer's behavior under different market structures (monopoly, duopoly,...) we need to decompose the problem; into two steps:
  - Compute the combination of inputs which minimizes cost *for a given output level*
  - Compute the optimal output chosen by the firm (under alternative assumptions on market structures)

Cost Minimization

- Let y be a fixed output target evel.
- What is the combination of inputs x_1 and x_2 which minimizes the cost of producing y?
- The problem is

$$\min_{x_1, x_2} w_1 x_1 + w_2 x_2 \text{ s to } f(x_1, x_2) = y.$$

- The answer to the problem is the *cost function*

$$C(y, w_1, w_2).$$

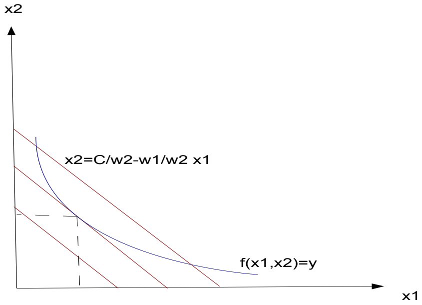
Graphical approach

- We compute the *isocost line* describing the combination of inputs x_1, x_2 which result in the same cost C:

$$x_2 = \frac{c}{w_2} - \frac{w_1}{w_2} x_1 .$$

- For a fixed y, we draw the isoquant such that $f(x_1, x_2) = y$
- We look at the point on the isoquant which corresponds to the lowest isocost line.
- This is the optimal input combination.

Cost Minimization



Characterization

- Cost is minimized at the point where

$$-\frac{MP_1}{MP_2} = TRS = -\frac{w_1}{w_2}$$

- *The technical rate of substitution is equal to the factor price ratio.*
- Consider a change in the pattern of production $(\Delta x_1, \Delta x_2)$ which leaves the level of production constant:

$$MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0.$$

- This change cannot lower cost,

$$w_1 \Delta x_1 + w_2 \Delta x_2 = 0.$$

- the opposite change $(-\Delta x1, -\Delta x_2)$ cannot decrease cost either so that

$$w_1 \Delta x_1 + w_2 \Delta x_2 = 0.$$

Characterization

- We conclude that

$$\frac{MP-1}{MP-2} = \frac{w_1}{w_2}.$$

Cost functions for perfect complements and perfect substitutes

- For perfect complements, $f(x_1, x_2) = \min(\{x_1, x_2\})$.
- The cost function is $C(y, w_1, w_2) = w_1 y + w_2 y$.
- For perfect substitutes $f(x, y) = x + y$.
- If $w_1 < w_2$, $C(y, w_1, w_2) = w_1 y$
- If $w_2 < w_1$, $C(y, w_1, w_2) = w_2 y$.

Cost function for a Cobb Douglas production function

- We write the Lagrangian

$$\mathcal{L} = w_1 x_1 + w_2 x_2 - \lambda(x_1^a x_2^b - y),$$

- and the first order conditions

$$\begin{aligned} w_1 - \lambda a x_1^{a-1} x_2^b &= 0, \\ w_2 - \lambda b x_1^a x_2^{b-1} &= 0, \\ x_1^a x_2^b &= y \end{aligned}$$

Cost function for the Cobb Douglas production function

- We first derive the factor demand functions

$$\begin{aligned} x_1(w_1, w_2, y) &= (\frac{a}{b})^{\frac{b}{a+b}} w_1^{-\frac{b}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}, \\ x_2(w_1, w_2, y) &= (\frac{b}{a})^{\frac{a}{a+b}} w_1^{-fracaa+b} w_2^{-\frac{a}{a+b}} y^{\frac{1}{a+b}}, \end{aligned}$$

- And obtain the cost function

$$C(y, w_1, w_2) = [(\frac{a}{b})^{\frac{b}{a+b}} + (\frac{b}{a})^{\frac{a}{a+b}}] w_1^{\frac{a}{a+b}} w_2^{\frac{b}{a+b}} y^{\frac{1}{a+b}}.$$

Revealed cost minimization

- Suppose that there are two sets of prices (w_1^s, w_2^s) and (w_1^t, w_2^t) and the associated choices (x_1^s, x_2^s), (x_1^t, x_2^t).
- Suppose that each of these choices produces the same output level y.
- By cost minimization

$$\begin{aligned} w_1^t x_1^t + w_2^t x_2^t &\leq w_1^t x_1^s + w_2^t x_2^s, \\ w_1^s x_1^s + w_2^s x_2^s &\leq w_1^s x_1^t + w_2^s x_2^t. \end{aligned}$$

- Adding up the two inequalities and rearranging

$$(w_1^t - w_1^s)(x_1^t - x_1^s) + (w_2^t - w_2^s)(x_2^t - x_2^s) \leq 0.$$

Revealed cost minimization

- We thus have

$$\Delta w_1 \Delta x_1 + \Delta w_2 \Delta x_2 \leq 0.$$

- If the price of factor 2 remains constant, the inequality implies

$$\Delta w_1 \Delta x_1 \leq 0.$$

- *Factor demand functions are decreasing in prices.*
- The cost function is *increasing in prices* and *increasing in output*

Returns to scale and cost

- If we have constant returns to scale, the cost function must be linear in output: in order to produce y units of output, we need to incur a cost equal to $c(w_1, w_2, 1)y$, where $c(w_1, w_2, 1)$ is the unit cost
- If returns to scale are increasing, $c(w_1, w_2, y) < c(w_1, w_2, 1)y$
- If returns to scale are decreasing, $c(w_1, w_2, y) > c(w_1, w_2, 1)y$.
- This can also be expressed in terms of *average cost*

$$AC(y) = \frac{C(y)}{y}.$$

- If returns to scale are increasing (decreasing constant), $AC(y) < C(1), (= C(1), > C(1))$.

Long run and short run costs

- The *short run cost function* is the cost function when one only adapts the variable factors of production (labor, intermediate goods)
- The *long run cost function* is the cost function when one can adjust all factors (including capital, land)
- In the short run, $x_2 = \overline{x_2}$.
- We compute the factor demand for factor 1, $x_1(w_1, y, \overline{x_2})$, and the short run cost

$$C_s(y, \overline{x_2}) = w_1 x_1(w_1, y, \overline{x_2}) + w_2(\overline{x_2}).$$

- Let the long run factor demands be $x_1(y)$, $x_2(y)$.
- Then we can establish a connection between short run and long run cost functions:

$$C(y) = C_s(y, x_2(y)).$$

Fixed, quasi fixed and sunk costs

- Fixed costs are costs which need to be paid independently of the level of output
- Quasi fixed costs are independent of the level of output but only paid if the firm produces
- Sunk costs are fixed costs which cannot be recovered (e.g. painting a new office, a plant with no resale value)

Average cost curve

- We decompose the cost into fixed and variable cost

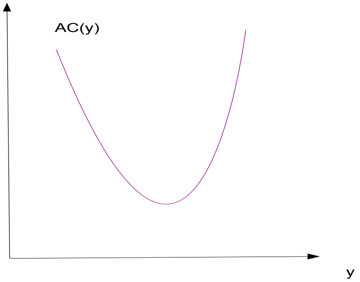
$$C(y) = C_v(y) + F.$$

- Average cost is

$$AC(y) = \frac{C_v(y)}{y} + \frac{F}{y}.$$

- Average fixed cost is decreasing with y ; average variable cost is increasing with y.
- The average cost is U shaped.

Average cost



Marginal costs

- The marginal cost is the additional cost incurred when producing one additional unit.

$$MC(y) = \frac{C(y + \Delta y) - C(y)}{\Delta y} = C'(y).$$

- Compute the derivative of $AC(y)$:

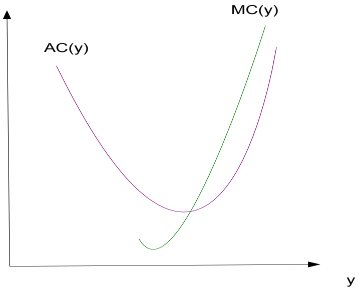
$$AC'(y) = \frac{C'(y)y - C(y)}{y^2}.$$

- When the average cost is decreasing, $MC(y) = C'(y) < \frac{C(y)}{y} = AC(y)$.
- When the average cost is increasing, $MC(y) = C'(y) > \frac{C(y)}{y} = AC(y)$.

Marginal costs

- The marginal cost is below average cost when average cost is decreasing, above average cost when average cost is increasing, equal to average cost at the minimum of average cost.

Marginal and average cost



Examples of cost curves

- Suppose that $C(y) = y^2 + F$.
- The average variable cost is $AVC(y) = y$
- The average fixed cost is $AFC(y) = \frac{1}{y}$
- The average cost is $AC(y) = y + \frac{1}{y}$
- The marginal cost is $MC(y) = 2y$.

Two plants

- Suppose there are two plants with different cost functions $C_1(y_1)$ and $C_2(y_2)$.
- How much should be reduced in each plant?

$$\min C_1(y_1) + C_2(y_2) \text{ s to } y_1 + y_2 = y.$$

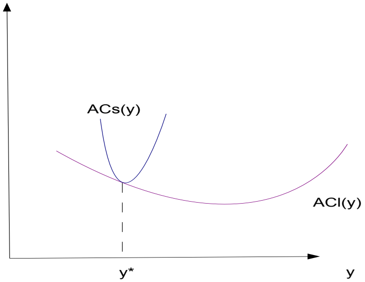
- At the optimum, the marginal cost of producing in each plant must be equal:

$$MC_1(y_1) = MC_2(y_2).$$

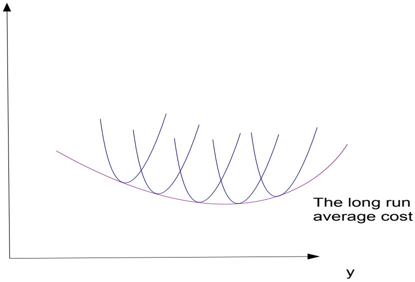
Long run costs

- For any given level of output y, there is an optimal plant size $k(y)$.
- In the short run, the output is y^* and the optimal plant size $k^* = k(y^*)$.
- Given a fixed k^*, the short run average curve is $AC_s(y) = \frac{C(y,k^*)}{y}$.
- In the long run, the firm adjusts plant size to the output level so the long run average curve is $AC_l(y) = \frac{C(y,k(y))}{y}$.
- The two curves meet at y^*, by definition, the long run average cost is always below the short run average cost.
- It is the *lower envelope* of the short run average cost curves.

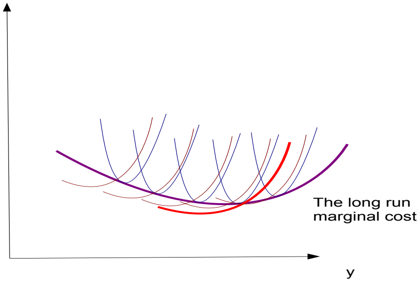
Long run and short run average costs



Long run average costs



Long run marginal costs



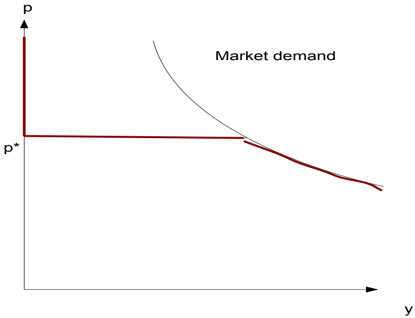
Market structures and supply

- We now want to compute, for any output price p, the optimal supply of the form $y(p)$
- In order to do this computation, we use the cost function $C(y)$
- The supply of the firm depends both on technological constraints (captured by the cost function) and on market constraints.
- There are three main market structures: pure competition, monopoly and oligopoly.

Pure competition

- In pure competition, every firm is small
- Firms are *price takers*: they view the market price p^* as fixed
- Even though there is a global market demand, firms perceive demand as follows
  - Above the market price p^* they cannot sell
  - Below the market price p^* they face the entire market demand.

Competitive demand



Market supply

- The firm chooses y to maximize
$$py - C(y),$$
- The solution is
$$p = C'(y) = MC(y).$$
- A competitive firm produces *at the point where price is equal to marginal cost.*
- This simple rule equates the additional revenue brought by an additional unit with the additional cost.
- If marginal revenue is higher than marginal cost, the firm should expand its production
- If marginal revenue is below marginal cost, the firm should reduce its

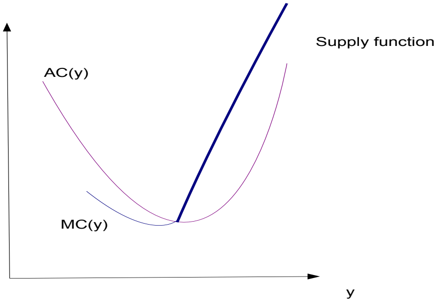
Shut down and break-even points

- The computation of optimal supply does not take into account fixed costs.
- The firm should also decide whether it is better off producing (with profit $py - C_v(y) - F$) or not (with profit 0)
- The *break even point* is the supply y^* at which

$$p = AC(y) = \frac{C_v(y)}{y} + \frac{F}{y}.$$

- If the competitive supply is below the break-even point, the firm is better off shutting down and not producing.
- This shows that the competitive supply follows the marginal cost curve above the minimum of average cost.

Supply function



The Supply function: An example

- Suppose that $C(y) = y^2 + 1$
- The average cost is $AC(y) = y + \frac{1}{y}$ which attains its minimum at $y = 1$ with an average cost $AC = 2$
- The marginal cost is $MC(y) = 2y$
- The supply curve is: produce 0 if $p < 2$, if $p \geq 2$ produce $y = \frac{p}{2}$.

Summary of Lecture VIII

- For a fixed output level y, we compute the optimal combination of inputs to produce y, $x_1(w_1, w_2, y)$ and $x_2(w_1, w_2, y)$
- Factor demand functions are increasing in output, decreasing in own factor price
- The cost function is increasing in factor prices and output
- Increasing returns to scale imply decreasing average cost, decreasing returns to scale increasing average cost and constant returns to scale constant average costs
- Costs can be fixed, quasi fixed and sunk if they cannot be recovered.
- Average costs are composed of variable and fixed average costs and are U shaped

Summary of Lecture VIII

- Marginal costs are below average costs when average cost is decreasing, above when average cost is increasing
- Marginal costs are equal to average costs at the minimum of average cost.
- The long run average cost is the lower envelope of short run average costs
- Firms supply decisions depend both on technological constraints and market structures
- In a purely competitive world, firms produce at the point where marginal cost is equal to price
- In order to cover fixed cost, the firm must produce above the break-even point
- the supply curve follows the marginal cost curve above the minimum of average cost.