# Lecture VII: Production

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# Technology

- We describe the behavior of producers, which supply goods.
- The firm is viewed as a ”black box” turning inputs into outputs.
- The theory of production is a ”flat balloon” in theory.
- Production theory is formally very similar to consumer theory.
- But in production theory, one observes the output (not in consumer theory)

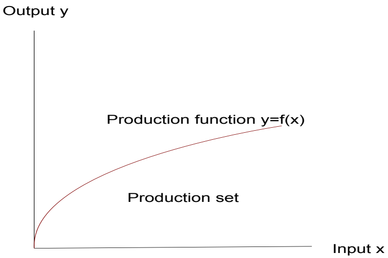
# Inputs and outputs

- Inputs: land, labor, capital, intermediate goods
- Outputs: single vs multiple outputs
- The production process describes how outputs are obtained from inputs¿

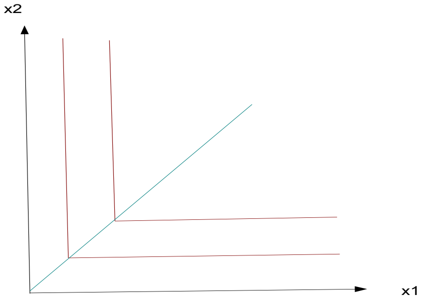
# Production functions

- A production set describes the set of inputs and outputs $(x, y)$ which are feasible.
- In a simple one input - one output case, we can focus on the *production function* which shows the maximum level of output $y$ obtained with input level $x$, $y = f(x)$
- If there are several inputs, we may write $y = f(x, y)$
- An *isoquant* in the space $(x_1, x_2)$ describes the set of combinations of $x_1$ and $x_2$ which result in the same output.
- Isoquants are similar to indifference curves.

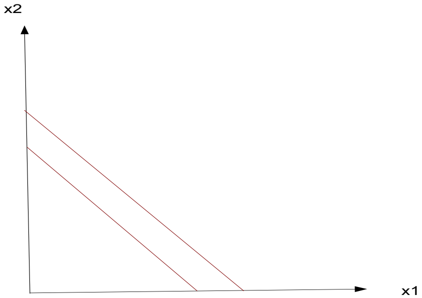
# Production set



# Leontieff technology



# Perfect substitutes



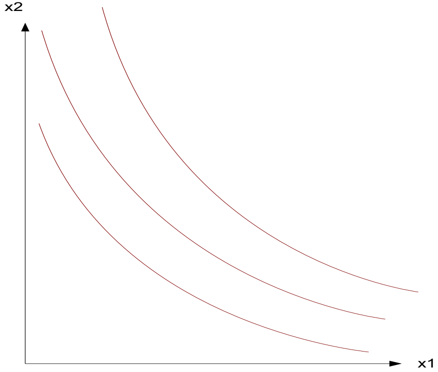
# Cobb Douglas technology

- The cobb Douglas technology is

$$y = f(x_1, x_2) = Ax_1^{\alpha} x_2^{\beta}.$$

- Both inputs are needed to be able to produce the output
- The coefficients $\alpha$ and $\beta$ measure the share of both inputs in the production.

# Cobb Douglas



# Properties of technology

- Technologies are *monotonic*: more inputs lead to more outputs, the production function is increasing.
- Technologies satisfy *free disposal*: the firm can costlessly dispose of inputs. So if $(x, y)$ is in the production set, any point $(x', y)$ with $x' < x, y' < y$ is also in the production set.
- Technologies are *convex*: If you can prouduce $y$ with $(x_1, x_2)$ and $(z_1, z_2)$, then any convex combination of $(x_1, x_2)$ and $(z_1, z_2)$ will result in at least as much output. Isoquants are convex.

# Marginal product

- The marginal product of input 1 at $(x_1, x_2)$ measures how much additional output can be produced by adding a little bit more of factor of production 1.
- We measure

$$\frac{\Delta y}{\Delta x} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}.$$

- When $f$ is differentiable,

$$MP_1 = \frac{\partial f}{\partial x_1}.$$

- The ;arginal product plays the same role as marginal utility in consumer theory.

# Technical rate of substitution

- The technical rate of substitution measures how much extra unit of factor 2 is needed to compensate a loss of factor 1 and keep the same level of production
- We compute

$$\Delta y = MP_1 \Delta x_1 + MP_2 \Delta x_2 = 0.$$

- We have

$$TRS_{12} = \frac{\Delta x_2}{\Delta x_1} = -\frac{MP_1}{MP_2}.$$

# Technologies long run and short run

- The *law of diminishing product* states that the marginal product of each input is decreasing.
- The *law of diminishing technical rate of substitution* states that the technical rate of substitution is decreasing
- In the short run, production technologies are fixed ; some production factors cannot be changed (e.g. land)
- In the long run, production technologies are flexible ; all levels of production can be changed.

# Returns to scale

- When we multiply both inputs by the same factor, how does the output change?
- With *constant returns to scale*, production is multiplied by the same factor (homogeneity of degree one)

$$f(\alpha x_1, \alpha x_2) = \alpha f(x_1, x_2).$$

- With decreasing returns to scale, $f(\alpha x_1, \alpha x_2) < \alpha f(x_1, x_2)$
- With increasing returns to scale, $f(\alpha x_1, \alpha x_2) > \alpha f(x_1, x_2)$

# Profit maximization

- Profits are revenus minus costs
- We assume that the markets for outputs and inputs are competitive.
- Prices of outputs are $p$
- Prices of input are $(w_1, w_2)$
- Profit is then:

$$\pi = py - w_1 x_1 - w_2 x_2.$$

# Costs and profits

- Costs are opportunity costs, not accounting costs
- Costs are flows: wages, rental rates
- Do firms maximize profit? (not necessarily: could maximize sales, wage bills..)
- the boundaries of a firm: outsourcing, firm as a nexus of contracts..

# Short run profit maximization

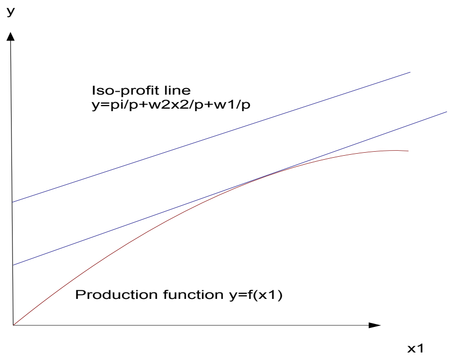
- In the short run, assume that one factor of production , $x_2$ is fixed at $\overline{x_2}$. The profit maximization problem becomes:

$$\max_{x_1} pf(x_1, \overline{x_2}) - w_1 x_1 - w_2 \overline{x_2}.$$

- The solution is

$$pMP_1 = w_1.$$

# Short run profit maximization



# Long run profit maximization

- In the long run, firms can choose both factors $x_1$ and $x_2$
- The conditions for profit maximization become:

$$\begin{aligned} pMP_1 &= w_1, \\ pMP_2 &= w_2 \end{aligned}$$

- We also use the feasibility constraint:

$$f(x_1, x_2) = y.$$

- The *factor demand functions* describe the demand for inputs 1 and 2 as a function of the input prices $w_1, w_2$ and the price of output $p$
- the demand for input $i$ is increasing in $p$ and decreasing in $w_i$

# Profit maximization with Cobb Douglas

- The marginal product conditions are:

$$\begin{aligned} pax_1^{a-1}x_2^b - w_1 &= 0, \\ pbx_1^a x_2^{b-1} - w_2 &= 0 \end{aligned}$$

- yielding:

$$\begin{aligned} x_1^* &= \frac{apy}{w_1}, \\ x_2^* &= \frac{bpy}{w_2} \end{aligned}$$

- The feasibility constraint is

$$x_1^{*a} x_2^{*b} = y,$$

- yielding $y^* = (\frac{pa}{w_1})^{\frac{a}{1-a-b}}(\frac{pb}{w_2})^{\frac{a}{1-a-b}}$

# Profit maximization and returns to scale

- Suppose that the profuction function exhibits constant returns to scale.
- The maximum profit is given by

$$\pi^* = pf(x_1^*, x_2^*) - w_1 x_1^* - w_2 x_2^*.$$

- Multiply both factors of production by $\alpha$ to get:

$$\begin{aligned} \pi &= pf(\alpha x_1^*, \alpha x_2^*) - w_1 \alpha x_1^* - w_2 \alpha x_2^*, \\ &= = \alpha pf(x_1^*, x_2^*) - w_1 \alpha x_1^* - w_2 \alpha x_2^*, \\ &= \alpha \pi^*. \end{aligned}$$

- If profit is maximized, it must be *equal to zero*
- If a firm has constant retunrs to scale, it makes zero profit!

# Productivity analysis

- Empirical challenge: measure productivity and changes in productivity
- Are changes in productivity due to a better utilization of resources or changes in technology?
- Separate *technological change efficiency* measured by movements of the production frontier, *production efficiency* measured by movements towards the current production frontier and *scale efficiency* measured by movements along the current production frontier.

# Productivity analysis in the single output - single input case

- In the single input - single output case, all the information about production is contained in a simple production function $y = f(x)$ and productivity is measured simply as the average product:

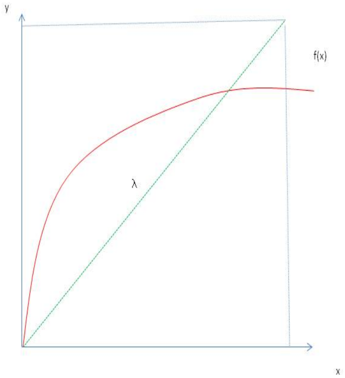
$$\text{MP} = \frac{y}{x},$$

- In order to measure efficiency, we compute for any pair of input output $(x, y)$ the *coefficient of resource utilization* (Debreu, 1953) as:

$$C(x, y) = \min\{\lambda | (x, \frac{y}{\lambda}) \leq f(x)\}.$$

- This coefficient gives a measure of the efficiency of the production process for every direction $(x, y)$.

# Coefficient of resource utilization



# The Fisher index

- We now generalize productivity measures to the $n$ output $m$ input cases.
- There are many possible productivity measures, as there are many ways to aggregate inputs and outputs.
- The *Fisher* productivity measure uses the *prices* of inputs $w$ and of outputs $p$ to define productivity. We thus define,

$$Y(y) = \sum_{i=1}^{n} p_i y_i, X(x) = \sum_{j=1}^{m} w_j x_j,$$

- and the productivity measure is :

$$MP(x, y) = \frac{Y(y)}{X(x)}.$$

# the Fisher index II

- When one measures the difference in productivity between two periods $t$ and $s$, one can either use as a basis for aggregation the prices of the initial period $(p^t, w^t)$ or the prices of the final period $(p^s, w^s)$.
- The *Fisher index* takes a geometric average of the two, and is defined by taking the aggregation functions:

$$
\begin{aligned}
Y(y) &= \sqrt{(\sum_{i=1}^{n} p_i^t y_i)(\sum_{i=1}^{n} p_i^s y_i)}, \\
X(x) &= \sqrt{(\sum_{j=1}^{m} w_j^t x_j)(\sum_{j=1}^{m} w_j^s x_j)},
\end{aligned}
$$

# The Tornqvist index

- the Tornqvist index uses as aggregators:

$$
\begin{aligned}
Y(y) &= \sum_{i=1}^{n} \frac{y_i}{\sum y_i} \log y_i, \\
X(x) &= \sum_{j=1}^{m} \frac{x_j}{\sum x_j} \log x_j.
\end{aligned}
$$

# Tornqvist and Fisher indices of productivity

| Year | Tornqvist | Fisher |
|---|---|---|
| 1987 | 100 | 100 |
| 1988 | 194.81 | 104.82 |
| 1989 | 104.87 | 104.87 |
| 1990 | 103.60 | 103.60 |
| 1991 | 101.28 | 101.28 |
| 1992 | 107.38 | 107.39 |

TABLE 1. TORQVIST AND FISHER INDICES OF US MANUFACTURING