Lecture III: Revealed Preferences, Price effects

Francis Bloch and Fabrice Le Lec[1]

[1]Université Paris I

October 13, 2014

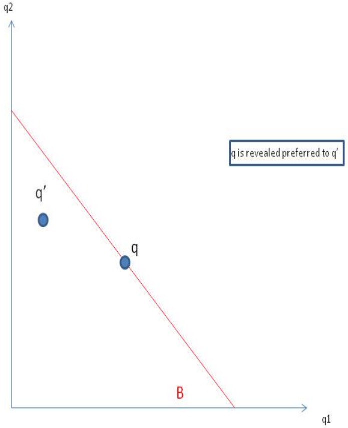
Revealed preference

- In reality, one does not observe directly preferences, but observes choices.
- The question becomes: how can we recover preferences from observing choices.
- Choices are given for different levels of prices p_1, p_2 and different income levels x.

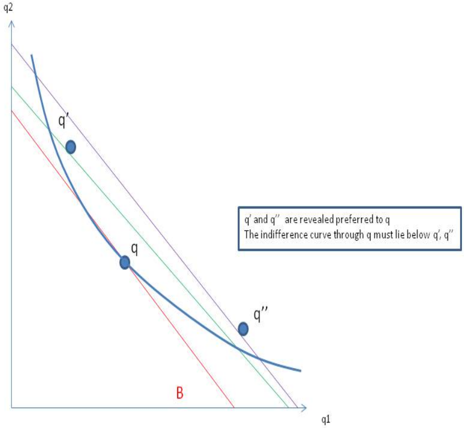
Revealed preference relation

- If, at some budget set B, two bundles q and q' are affordable, and the consumer chooses q, the bundle q is revealed preferred to q', i.e. $qRPq'$.
- In fact, all bundles which are in B are revealed lower than bundle q.

Revealed preference I



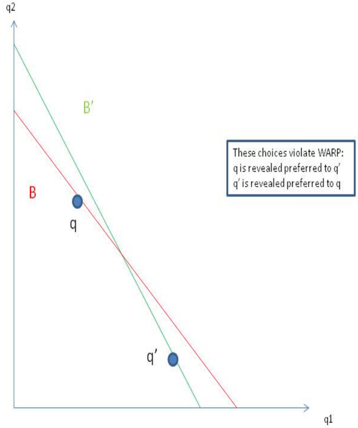
Recovering indifference curves from choices



The weak axiom of revealed preference

- In order for choices to be consistent (based on rational preferences), we need the following to be true:
- *Weak axiom of revealed preference*: If q is revealed preferred to q', then q' is not revealed preferred to q.

WARP


q2
B'
B
q
q'
q1
These choices violate WARP:
q is revealed preferred to q'
q' is revealed preferred to q

Checking WARP

Observation	p_1	p_2	q_1	q_2	bundle
1	1	2	1	2	1
2	2	1	2	1	2
3	1	1	2	2	3

Checking WARP II

Observation	cost of 1	cost of 2	cost of 3
1	5	4^*	6
2	4^*	5	6
3	3^*	3^*	4

- Stars show that: bundle 1 is revealed preferred to 2, bundle 2 revealed preferred to 1 (contradiction) and bundle 3 revealed preferred to 1 and 2.

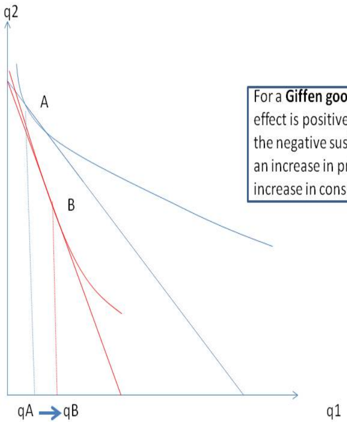
Indirect revealed preference and SARP

- We define the *transitive closure* of the (direct) revealed preference relation: If q is revealed preferred to q' and s' revealed preferred to q'' then q is revealed preferred to q''. This (indirect) revealed preference relation is *stronger* than the direct preference relation: it ranks more bundles.
- We can strengthen WARP into the *Strong Axiom of Revealed Preference.* If q is indirectly revealed preferred to q', it cannot be that s' is indirectly revealed preferred to q.

Price effects

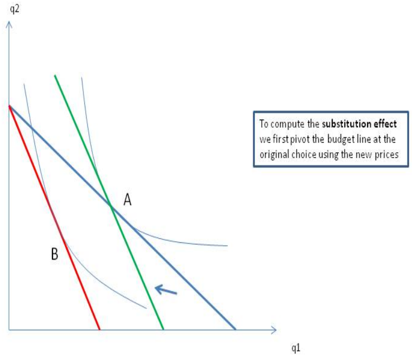
- When the price of a good increases, the consumption of that good may either decrease or increase (case of Giffen goods)
- The Slutsky decomposition is a formula which helps us understand the effect of prices of consumption
- It is based on a decomposition between a substitution and an income effect.
- To do this decomposition, we first keep income constant, but change relative prices and then keep increase income.

Giffen goods



For a **Giffen good**, the income effect is positive and overcomes the negative susbtitution effect: an increase in price results in an increase in consumption

Pivoting the budget line



Money compensation

- At the original choice we have: $p_1 q_1 + p_2 q_2 = x$.
- As the price of good 1 has changed, what is the income needed to afford the original consumption A?

$$p_1' q_1 + p_2 q_2 = x'.$$

- The difference in income is thus given by:

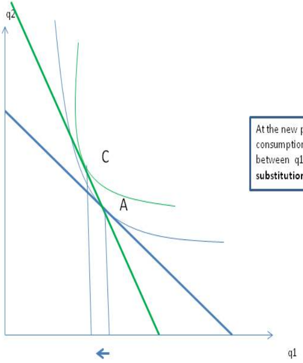
$$\Delta x = q_1 \Delta p_1.$$

Substitution effect

- If we keep purchasing power constant, and change price to p_1' and income to x', the optimal consumption of the agent will switch from A to B.
- The *substitution effect* measures the difference in the consumption of good 1 in A and C:

$$\Delta q_1^s = g_1(p_1', x') - g_1(p_1, x).$$

The substitution effect



At the new prices p',x'', the optimal consumption is at C: the difference between q1 at A and C is the **substitution** effect.

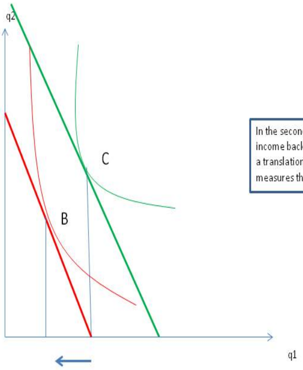
The income effect

- In a second step, we analyze what happens when the income moves from x' to x.
- The income effect is the difference

$$\Delta x_1^n = x_1(p_1', x) - x_1(p_1', x').$$

- If the price increases, x' is greater than x, so the move corresponds to a decrease in income.
- If the good is normal, this implies that Δx_1^n is negative, but if the good is inferior, Δx_1^n can be positive.
- For a Giffen good, the negative income effect is higher than the substitution effect.

The income effect



In the second step, we adjust income back : this corresponds to a translation of the budget line and measures the **income effect**

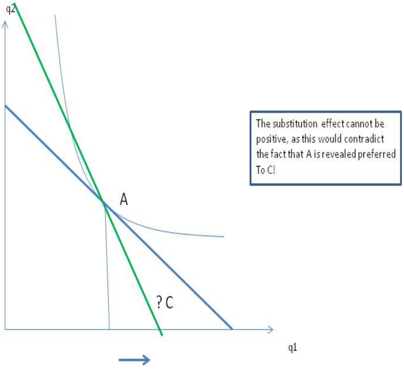
The Slutsky decomposition

- Named after the Russian economist E Slutsky:

$$\Delta q_1 = \Delta q_1^s + \Delta q_1^n.$$

- The substitution effect is *always negative*
- The income effect is either negative (if the good is normal) or positive (if the good is inferior)

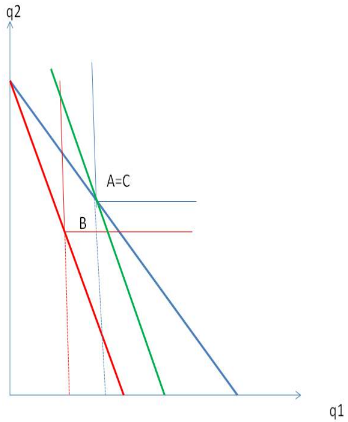
The substitution effect must be negative



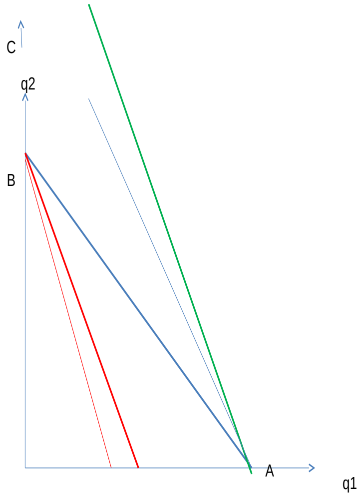
The law of demand

- If the demand for a good increases when income increases, then the demand for that good must decrease when price increases.
- If the goods are perfect complements, there is only an income effect.
- If the goods are perfect substitutes, there is only a substitution effect.

Slutsky decomposition with perfect complements



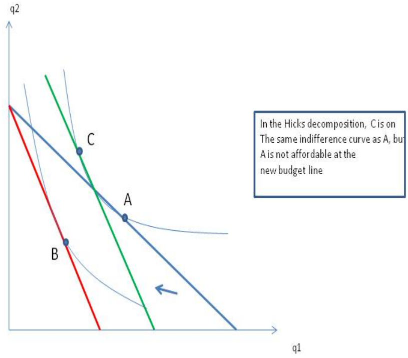
Slutsky decomposition with perfect substitutes



The Hicks decomposition

- In the Hicks decomposition, the substitution effect is computed not by pivoting the budget line at the initial choice but around the indifference curve through the original choice:
- The point *C* is the point tangent to the indifference curve through *A* at the new relative prices
- The price effect is then again decomposed into a substitution and an income effect.

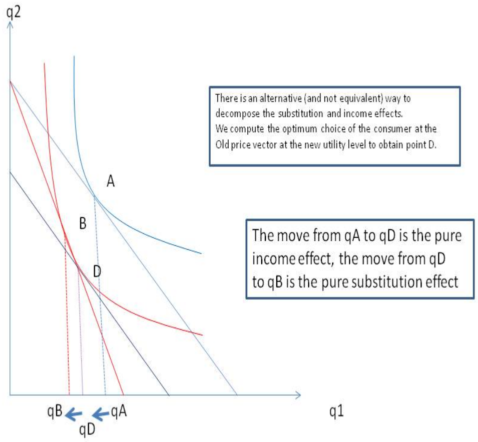
The Hicks decomposition



Alternative decompositions

- Both in the Slutsky and Hicks decompositions, we use as a basis the optimal choice at the original price p_1, and compute the substitution effect by pivoting the budget line at that point.
- An alternative is to use as a basis the optimal choice at the final price p_1' and pivot the budget line at that point.
- This alternative decomposition will result in a different computation of the substitution and income effects.

Hicks decomposition based on final choice



From individual to market demand

- Marshallian demands derive from individual maximization, and are individual demand functions.
- In order to generate *market demand*, we add the individual demands of all consumers on a market.
- When do market demand behave like Marshallian demands? (i.e. when is it the case that goods are normal, inferior at the market level, when is it possible to derive the effect of prices on consumption?)
- In order to answer this question, we ask: when can market demand be expressed as a function of prices and the *sum of individual incomes?*

Aggregation of demand

- If preferences are homothetic, Marshallian demands are *linear* in income.
- Hence, the sum of demands only depends on the sums of income $\Rightarrow$ **aggregation of demands is exact**
- Conversely, aggregation of demands is exact only if Marshallian demands are linear in income.
- (This is a property of homothetic preferences, but other preferences satisfy it as well, e.g. pure substitutes or complements)

Demand curve

Most of the time, we draw the **inverse demand curve**, putting prices on the y axis and quantities on the x axis.

Price elasticity

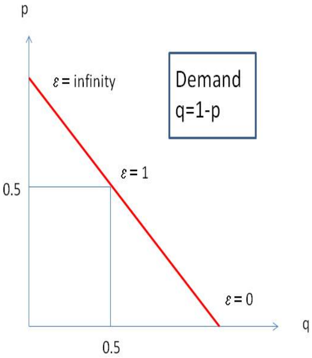
- We want to compute (get a quantitative estimate) the effect of a change in price on demand
- Just computing the difference in demand $g_i(x, p') - g_i(x, p)$ is not enough: this depends on the change $p' - p$.
- just computing the rate at which demand increases $\frac{g_i(x,p') - g_i(x,p)}{p' - p}$ is not enough: it depends on the units in which price and demand are measured.

Price elasticity II

-
- In order to obtain a measure which is independent of units, we compute how a one percent change in price affects the demand of the good in percentage: $\epsilon_i(p,x) = \frac{\frac{g_i(x,p_i')-g_i(x,p)}{g_i(x,p)}}{\frac{p_i'-p_i}{p_i}}$.
- This measure is called the *elasticity of demand with respect to price.*
- If demand is differentiable, we can compute it more simply as:

$$\begin{aligned}\epsilon_i(p,x) &= \frac{\partial g_i(p,x)}{\partial p_i}\frac{p_i}{g_i(p,x)} \\ &= \frac{\partial \log g_i(p,x)}{\partial p_i}.\end{aligned}$$

Price elasticity along a linear demand curve



Income elasticity

- We compute how a one percent change in income affects the demand of the good in percentage:
  $\eta_i(p,x) = \frac{\frac{g_i(x,p)-g_i(x',p)}{g_i(x,p)}}{\frac{x-x'}{x}}$.
- This measure is called the *elasticity of demand with respect to income.*
- If demand is differentiable, we can compute it more simply as:

$$\begin{aligned}\eta_i(p,x) &= \frac{\partial g_i(p,x)}{\partial x}\frac{x}{g_i(p,x)} \\ &= \frac{\partial \log g_i(p,x)}{\partial x}.\end{aligned}$$

Income elasticity: normal and inferior goods

- If the good is inferior, $\eta_i(p, x) < 0$
- If the good is normal, $\eta_i(p, x) > 0$
- If the good is a luxury good, $\eta_i(p, x) > 1$.
- If the good is a necessity, $1 > \eta_i(p, x) > 0$.

Identity on elasticities

- Consider a change in income, and compute the new consumptions of two goods. We have

$$p_1 \Delta q_1 + p_2 \Delta q_2 = \Delta x.$$

- Divide by income:

$$\frac{p_1 q_1}{x} \frac{\Delta q_1}{q_1} + \frac{p_2 q_2}{x} \frac{\Delta q_2}{q_2} = \frac{\Delta x}{x}.$$

- Let $s_i = \frac{p_i q_i}{x}$ denote the *expenditure share of good i*
- Divide the equation by $\frac{\Delta x}{x}$:

$$s_1 \eta_1(p, x) + s_2 \eta_2(p, x) = 1.$$

- The weighted average of income elasticities is equal to 1.

Summary of Lecture III

- In order to elicit preferences, we observe consumption choices at different prices and income levels.
- Revealed preference theory enables us to construct preferences
- In order to be rational, choices must satisfy the weak and strong axiom of revealed preferences
- The effect of a change in price is decomposed into a substitution and income effect
- The substitution effect is always negative, the income effect is negative if the good is normal, positive if the good is inferior.

Summary of Lecture III

- The Slutsky equation describes the substitution and income effects.
- There are two decompositions: the Slutsky and Hicks decompositions
- To obtain market demand, one adds up individual demands.
- Demand can be represented by a representative agent if and only if preferences are homothetic.
- Elasticities measure the effect of changes in income and prices on demand.