Lecture II: Demand functions

Francis Bloch and Fabrice Le Lec[1]

[1] Université Paris I

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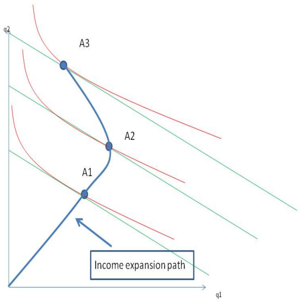
Marshallian demand functions

- By maximizing utility over the budget set, the consumer chooses the quantity consumed of each good i, $g_i$as a function of the prices p and the income level x.
- This is the *Marshallian demand function*.
- We first note that the Marshallian demand function is *homogeneous of degree zero* in prices and income: if prices and income are multiplied by the same factor λ, the demand function does not change: $g_i(\lambda p, \lambda x) = g_i(p, x)$.
- This property stems from the fact that the budget set is the same for $\lambda p, \lambda x$ and for p, x.
- It reflects the *absence of money illusion:* whether prices are quoted in euros or dollars does not change consumption.

Effect of income on consumption

- We first analyze how changes in income affect consumption:
- If x changes, the entire vector of consumption changes (this is the "income expansion path")
- If x changes, the consumption of every good i changes (this is the Engel curve of good i)
- If x increases, at least one of the consumptions must increase. It cannot be that all consumptions go down when income increases!

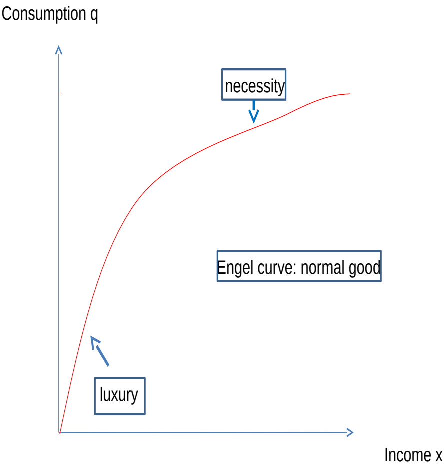
Income expansion path



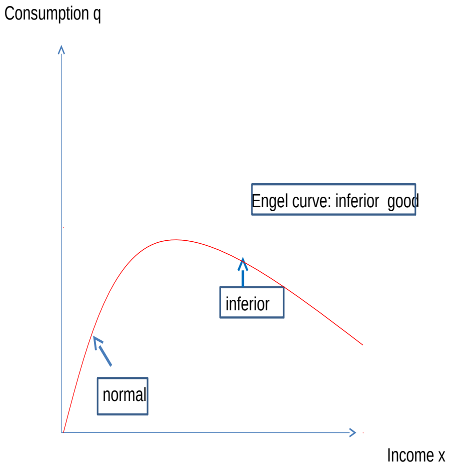
Normal and inferior goods

- A good is called *normal* if its consumption increases with income
- A good is called *inferior* if its consumption decreases with income
- Among normal goods, we distinguish between *luxury goods* whose consumption increases faster than income, and *necessities* whose consumption increases more slowly than income.

Engel curve normal good



Engel curve inferior good



Income elasticity

- We want to compute (get a quantitative estimate) the effect of a change in income on consumption
- Just computing the difference in consumption $g_i(x, p) - g_i(x', p)$ is not enough: this depends on the change $x - x'$.
- just computing the rate at which consumption increases $\frac{g_i(x,p)-g_i(x',p)}{x-x'}$ is not enough: it depends on the units in which income and consumption are measured.

Income elasticity II

- In order to obtain a measure which is independent of units, we compute how a one percent change in income affects the consumption of the good in percentage:

$$\eta_i(p,x) = \frac{\frac{g_i(x,p)-g_i(x',p)}{g_i(x,p)}}{\frac{x-x'}{x}}.$$

- This measure is called the *elasticity of consumption with respect to income.*
- If demand is differentiable, we can compute it more simply as:

$$\begin{aligned} \eta_i(p,x) &= \frac{\partial g_i(p,x)}{\partial x}\frac{x}{g_i(p,x)} \\ &= \frac{\partial \log g_i(p,x)}{\partial x}. \end{aligned}$$

Income elasticity: normal and inferior goods

- If the good is inferior, $\eta_i(p, x) < 0$
- If the good is normal, $\eta_i(p, x) > 0$
- If the good is a luxury good, $\eta_i(p, x) > 1$.
- If the good is a necessity, $1 > \eta_i(p, x) > 0$.

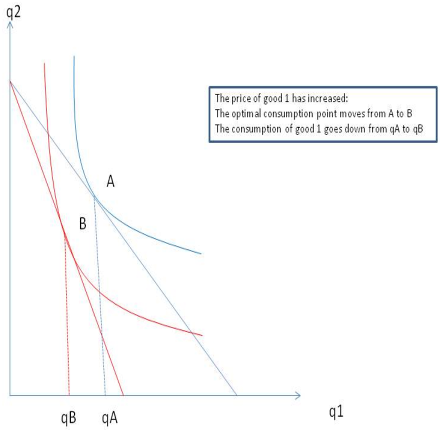
Empirical estimation of Engel curves

- Estimate the relation: $\frac{g_i(p,x)}{x} = \alpha_i + \beta_i \log x$
- Results of a 1974 British survey
- For food, drink, tobacco $\alpha = 0.947, \beta = -0.164$
- For housing and fuel $\alpha = 0.492, \beta = -0.077$
- For clothing and durables, $\alpha = -0.180, \beta = 0.092$
- For services, transport $\alpha = -0.259, \beta = 0.149$

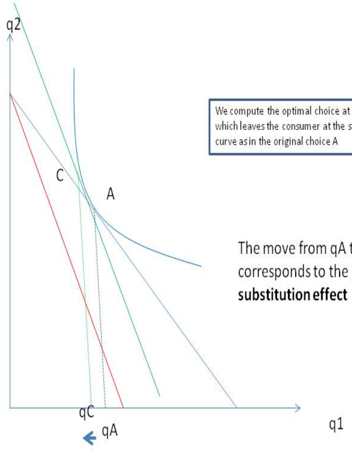
Effects of a change in price: substitution and income effects

- We are also interested in the effect of a change in price p_i on the consumption of good i
- This own-price effect is usually assumed to be *negative*, but the answer is complex.
- If the price p_i goes up, there are two effects:
- *a substitution effect* which is always negative and leads to a decrease in the consumption of good i
- *an income effect* which is negative if the good is normal, but may be positive if the good is inferior!

Effect of a price increase



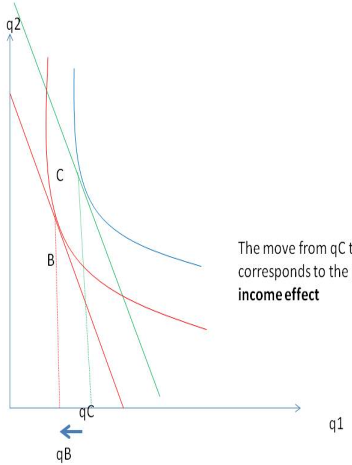
Decomposition: the pure substitution effect



We compute the optimal choice at the new price which leaves the consumer at the same indifference curve as in the original choice A

The move from qA to qC corresponds to the pure **substitution effect**

Decomposition: the pure income effect

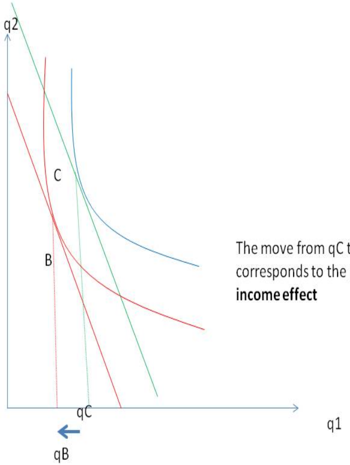


The move from qC to qB corresponds to the pure **income effect**

Income and substitution effects

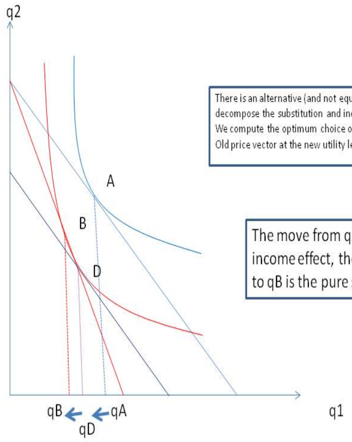
- The substitution effect is always negative
- The income effect is either negative (normal good) or positive (inferior good)
- If the income effect is positive and greater in absolute value than the substitution effect, an increase in price may result in an increase in demand. This is called a *Giffen good*

Giffen goods



The move from qC to qB corresponds to the pure **income effect**

An alternative decomposition



There is an alternative (and not equivalent) way to decompose the substitution and income effects.
We compute the optimum choice of the consumer at the Old price vector at the new utility level to obtain point D.

The move from qA to qD is the pure income effect, the move from qD to qB is the pure substitution effect

Price elasticity

- In order to quantify the effect of prices on demand, we compute price elasticity as

$$\epsilon_i(p,x) = \frac{\frac{g_i(p_i,pj,x)-g_i(p_i',p_j,x)}{g_i(p_i,p_j,x)}}{\frac{p_i-p_i'}{p_i}}.$$

- When the demand function is differentiable,

$$\begin{aligned}\epsilon_i(p,x) &= \frac{\partial g_i(p,x)}{\partial p_i}\frac{p_i}{g_i(p,x)} \\ &= \frac{\partial \log g_i(p,x)}{\partial p_i}.\end{aligned}$$

- In the usual case (when the good is not a Giffen good), the demand function is decreasing and $\epsilon_i(p,x) < 0$.

Cross price effects

- What is the effect of an increase in the price p_j on the demand of good i?
- In general, that effect can either be positive or negative.
- If an increase in the price p_j increases the consumption of good i, the goods i and j are *substitutes*
- If an increase in the price p_j reduces the consumption of good i, the goods i and j are *complements*

Cross price elasticities

- We compute cross price elasticity as:

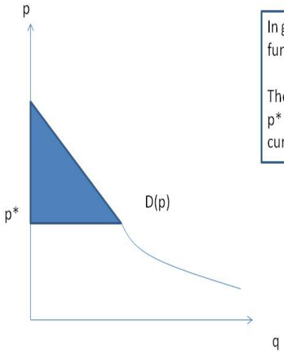
$$\epsilon_{ij}(p,x) = \frac{\frac{g_i(p_i,p_j,x)-g_i(p_i,p_j',x)}{g_i(p_i,p_j,x)}}{\frac{p_j-p_j'}{p_j}}.$$

- When the demand function is differentiable,

$$\begin{aligned}\epsilon_{ij}(p,x) &= \frac{\partial g_i(p,x)}{\partial p_j}\frac{p_j}{g_i(p,x)} \\ &= \frac{\partial \log g_i(p,x)}{\partial p_j}.\end{aligned}$$

- If the two goods are substitute, $\epsilon_{ij} > 0$; if the two goods are complements $\epsilon_{ij} < 0$.

Consumer surplus



In general, the demand function D(p) is decreasing.

The **consumer surplus** at price p* is the area below the demand curve D(p) and above the price p*

Consumer surplus

- If the consumer faces a price p^*, she buys all units at that price.
- Some units are valued higher than p^*. These are all the units on the demand curve above p^*.
- The consumer ”gains” on all those units. The sum of these gains is the *consumer surplus*: the area above price p^* and below the demand curve $D(p)$.

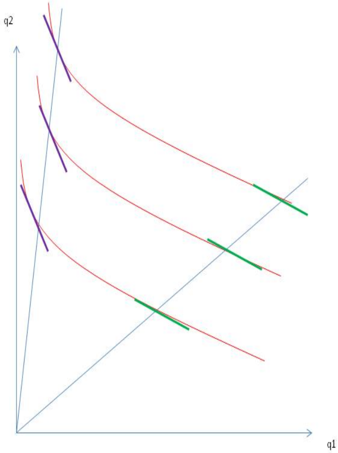
From individual to market demand

- Marshallian demands derive from individual maximization, and are individual demand functions.
- In order to generate *market demand*, we add the individual demands of all consumers on a market.
- When do market demand behave like Marshallian demands? (i.e. when is it the case that goods are normal, inferior at the market level, when is it possible to derive the effect of prices on consumption?)
- In order to answer this question, we ask: when can market demand be expressed as a function of prices and the *sum of individual incomes?*

Homothetic preferences

- Homothetic preferences are a very important special class of preferences/utility functions.
- A preference is homothetic if along any ray, the marginal rate of substitution between any pair of goods remains constant, or
- $MRS_{ij}(\lambda q_1, \lambda q_2) = MRS_{ij}(q_1, q_2)$ for any q_1, q_2, λ
- Alternatively, we can state: the Marginal Rate of Substitution is homogeneous of degree zero.
- The Cobb Douglas and Constant Elasticity of Substitution utilities are homothetic, but not Stone Geary.

Homothetic preferences



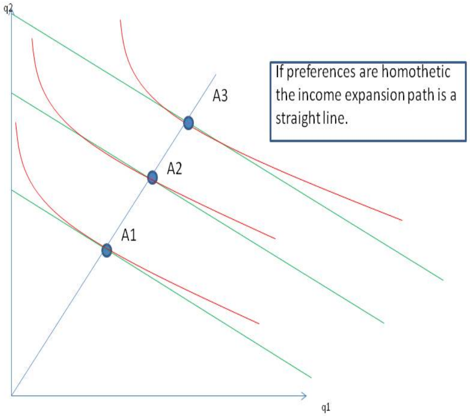
Income effects with homothetic preferences

- Consider two income levels x and $x' = \lambda x$.
- Let (q_1^*, q_2^*) be the optimal consumptions at x.
- This implies that:

$$\begin{aligned} MRS_{12}(q_1^*, q_2^*) &= \frac{p_1}{p_2} \\ p_1 q_1^* + p_2 q_2^* &= x. \end{aligned}$$

- **If preferences are homothetic, then $(\lambda q_1^*, \lambda q_2^*)$ is the optimal choice at $x' = \lambda x$.**
- Note that, because preferences are homothetic, $MRS_{12}(\lambda q_1^*, \lambda q_2^*) = MRS_{12}(q_1^*, q_2^*) = \frac{p_1}{p_2}$ and $p_1 \lambda q_1^* + p_2 \lambda q_2^* = \lambda x = x'$ so that the optimality conditions hold.

Income expansion path for homothetic preferences



Aggregation of demand

- If preferences are homothetic, Marshallian demands are *linear* in income.
- Hence, the sum of demands only depends on the sums of income $\Rightarrow$ **aggregation of demands is exact**
- Conversely, aggregation of demands is exact only if Marshallian demands are linear in income.
- (This is a property of homothetic preferences, but other preferences satisfy it as well, e.g. pure substitutes or complements)

Indirect utility

- Define indirect utility (as a function of p and x) as the maximal utility,

$$V(x,p) = u(q^*) = \max_{q|pq \leq x} u(q),$$

- Indirect utility is homogeneous of degree zero ,$V(\lambda x, \lambda p) = V(x,p)$ for all $\lambda > 0$
- If x increases, the budget set increases so that indirect utility is *increasing* in x.
- If any price p_i increases,t he budget set is reduced so that indirect utility is *decreasing* in p_i.

Summary of lecture II

- The Marshallian demand is a function of income and prices
- Goods are normal (inferior) if their consumption increases (decreases) with income
- The income expansion path maps the increase in consumption when income increases
- The Engel curve of a good measures its consumption as a function of income.
- If a price increases, it has two effects on consumption: a substitution effect (negative) and an income effect (negative or positive)
- If the income effect is positive and dominates the substitution effect, and increase in price will increase consumption: this is the case of a Giffen good.
- Effects of income and prices on consumption are measured by elasticities.

Summary of lecture II cont

- The consumer surplus is the area below the demand curve above a given price: it measures the gains of consumers.
- Preferences are homothetic if the MRS is homogeneous of degree zero. For homothetic preferences, the income expansion path is a straight line.
- Individual demands can be aggregated into market demands only if demands are linear in income (which happens when preferences are homothetic)
- The indirect utility function measures the maximal level of utility obtained for fixed prices and income. It is homogeneous of degree zero, increasing in income and decreasing in prices.