Patent races

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The timing of innovations

- Since Schumpeter, innovations have been seen as the engine of growth.
- We want to understand the *timing* of innovations.
- Who invests how much and when?
- How do sequences of innovations emerge?
- How is innovation affected by changes in the regulatory environment (patent protection)?
- How are innovations adopted and diffused in an industry?

Deterministic and stochastic models

- In a deterministic model, the firms invest a fixed amount in R& D and the firm with the highest amount wins (all-pay auction)
- See Dasgupta Stiglitz (1980), Gilbert and Newbery (1982)
- In a stochastic model, firms' investments are mapped into a probability of obtaining the innovation at each period
- See Loury (1979), Lee and Wilde (1980) and Reinganum (1982)

The social value of innovation

- We consider a cost innovation, resulting in a reduction from constant marginal cost c to c^*
- Social surplus is given by $s(c) = max_Q u(Q) - cQ$
- Using the envelope theorem,

$$s(c^*) - s(c) = \int_{c^*}^{c} \frac{\partial s}{\partial c} dc = \int_{c^*}^{c} Q_s(c) dc.$$

- We write the instantaneous social benefit as $\pi_s = s(c^*) - s(c)$ and the value as $V_s = \frac{\pi_s}{r}$.

The monopoly's value of innovation

- The profit of the monopoly is $\Pi(c) = \max_Q P(Q)Q - cQ$
- And
$$\Pi(c^*) - \Pi(c) = \int_{c^*}^{c} \frac{\partial \Pi}{\partial c} dc = \int_{c^*}^{c} Q_m(c) dc.$$
- As the monopoly's quantity is always below the socially optimal quantity, $\pi_m < \pi_s$, the monopoly's value of innovation is below the social value.
- We also define $V_m = \frac{\pi_m}{r}$.

The entrant's value of innovation

- If an entrant obtains the innovation, she obtains monopoly profit during the lifetime of the patent.
- The entrant sets a price at the cost c of the incumbent and obtains a profit $\Pi^e = (c - c^*)Q_e(c)$
- We again compute the flow value of the innovation as $\pi_e = \int_{c^*}^{c} Q_e(c)dc$.
- As $Q_e > Q_m$ and $Q_s > Q_e$, $\pi_s > \pi_e > \pi_m$
- The value of the innovation is $V_e = \frac{\pi_e}{r}(1 - e^{-rT})$ where T is the patent's length.

Research investments

- We suppose that by investing x, one obtains the innovation at the deterministic time $T(x)$.
- $T'(x) < 0$ and $T''(x) > 0$ (diminishing returns to investment)
- We also have $T(0) = \infty$ and $T(\infty) = 0$.

Socially optimal investment

- Maximize $V_s e^{-rT(x)} - x$
- This is in general not a concave function! First order conditions do not help.

The optimal investment of a monopolist

- Maximize $V_m e^{-rT(x)} - x$
- As V_m, V_s, the marginal benefit of the investment is lower under monopoly than the social optimum.
- As the marginal cost is the same, $x_m < x_s$: the monopoly invests too little in R & D.

Competition among firms

- Suppose that firms simultaneously choose their investment level x_i
- The winning firms cannot make positive profit, as any other firm could 'undercut' it by choosing a higher investment by a small amount ϵ
- There cannot be an equilibrium where a losing firm participates, as investment is costly.
- *As in an all pay auction with complete information there is no pure strategy equilibrium, all equilibria are in mixed strategy.*
- Dasgupta and Stiglitz focus on an 'alternative' equilibrium where one firm invests at the point where

$$V_e e^{-rT(x)} = x,$$

and the other firms refrain from investing.

Comparative statics and comparisons

- In a competitive market, average revenue is equal to zero (as opposed to marginal revenue in the case of monopoly and social optimum). This should result in excessive spending in competition.
- When the patent length T increases, π_e increases and so does x_e
- The comparison between x_e and x_m depends on the comparison between π_e and π_m. This in turn depends on demand elasticity.

The persistence of monopoly

- Suppose that one firm has a monopoly on a product, and a new innovation may infringe on this monopoly power
- Example:successive versions of computers, software..
- Who has the highest incentive to do research? The monopoly to protect its profit, or the new entrant to start making profit?

The model

- There are two products: the current product on which firm 1 has a monopoly, and sells at price P_1^m and a new product sold (either by the monopoly or entrant) at a price P_2
- Before patenting the monopoly has a profit $\Pi(P_1^m)$
- After patenting by the monopoly, profits are $\Pi(P_1^m, P_2^m)$ and 0
- After patenting by the entrant, profits are $\Pi^m(P_1^m, P_2^e)$ and $\Pi^e(P_1^m, P_2^e)$

Competition among entrants

- Suppose that potential entrants spend money to enter the market.
- As in Dasgupta Stiglitz, discovery time is deterministic, and let $C(T)$ be the cost of research allowing to obtain discovery at date T
- In a competition among entrants, discovery time will be the earliest date at which zero profit is made:

$$C(T^*) = \int_{T^*}^{\infty} \Pi^e(P_1^m, P_2^e)e^{-rt}dt.$$

- The monopoly has an expected gain of

$$\int_0^{T^*} \Pi(P_1^m)e^{-rt}dt + \int_{T^*}^{\infty} \Pi^m(P_1^m, P_2^e)e^{-rt}dt.$$

Investment by the monopoly

- If the monopoly invests $C(T)$, he obtains the innovation and obtains a profit

$$\int_0^{T^*} \Pi(P_1^m)e^{-rt}dt + \int_{T^*}^{\infty} \Pi^m(P_1^m, P_2^m)e^{-rt}dt - C(T^*).$$

- Hence the monopolist prefers to invest if

$$\int_{T^*}^{\infty} \Pi^m(P_1^m, P_2^m)e^{-rt}dt - C(T^*) \geq \int_{T^*}^{\infty} \Pi^m(P_1^m, P_2^e)e^{-rt}dt,$$

- Or

$$\int_{T^*}^{\infty} [\Pi^m(P_1^m, P_2^m) - \Pi^m(P_1^m, P_2^e) - \Pi^e(P_1^m, P_2^e)]e^{-rt}dt \geq 0.$$

Who invests more?

- The monopoly invests more (and hence persists) if

$$\Pi^m(P_1^m, P_2^m) - \Pi^m(P_1^m, P_2^e) \geq \Pi^e(P_1^m, P_2^e).$$

- The loss in profit due to the entry is higher than the profit of the entrant.
- This model shows that the monopoly may invest to maintain its position
- This can also be done by using sleeping patents, or investing in capacity to change the profits of the entrant/monopoly after patenting.

The stochastic model

- The first firm to innovate obtains a fixed value V
- Belated innovators do not obtain anything.
- Each firm i invests a fixed amount x_i at the beginning of the game.
- This investment translates into a probability distribution over the dates τ at which the product is found with

$$\Pr[\tau \leq t] = 1 - e^{-h(x_i)\tau},$$

- τ is exponentially distributed with expected time $\frac{1}{h(x_i)}$.

The stochastic model II

- $h(0) =) = \lim_{x \to \infty} h'(x)$,
- $h''(x) \geq (\leq) 0$ as $x \leq (\geq) \overline{x}$,
- Let $\hat{\tau}_i$ be the time at which the innovation is found by one of the firms other than i. Using the properties of the exponential function, with $a_i = \sum_{j \neq i} h(x_j)$

$$\Pr[\hat{\tau}_i \leq t] = 1 - e^{a_i t}.$$

- At any date t,

$$\begin{aligned} \Pr[\tau_i \leq t] &= e^{-a_i t}(1 - e^{-h(x_i)t}) \\ &+ e^{-h(x_i)t} \int_o^t a_i e^{-a_i s}(1 - e^{-h(x_i)}) ds, \\ &= \frac{h(x_i)}{a_i + h(x_i)}[1 - e^{-t(a_i + h(x_i))}]. \end{aligned}$$

Best responses

- Given a_i, r and V, firm i chooses x_i to maximize

$$\int_0^{\infty} V \Pr[\tau_i \leq \min\{t, \hat{\tau}_i\}] e^{-rt} dt - x_i,$$

- or

$$U = \frac{Vh(x_i)}{r(a_i + r + h(x_i))} - x_i,$$

- yielding a best response function:

$$\frac{h'(x)(a + r)}{(a + r + h(x))^2} = \frac{r}{V}.$$

- x is increasing in v, decreasing in r.

Equilibrium

- As firms are symmetric, equilibrium is given by

$$\frac{(n-1)h(x)+r)h'(x)}{(r+nh(x))^2}=\frac{r}{V}.$$

- *Result 1.* The equilibrium investment, x is decreasing in n.
- *Result 2.* Suppose that a marginal increase in R & D investment by any single firm causes the investment of each other firm to fall by a smaller amount. Then the expected time of discovery is decreasing in n.

Expected profit and long run equilibrium

- The expected profit of a firm is given by

$$\frac{h(x)}{h'(x)} \frac{nh(x) + r}{(n-1)h(x) + r} - x.$$

- *Result 3.* If the technology of innovation exhibits decreasing returns to scale throughout ($h'' < 0$), then expected profits fo to zero only at the limit when n goes to infinity.
- *Result 4.* With initial increasing returns to scale, free entry eventually drives expected profits to zero even for a finite number of firms.

Welfare analysis

- One source of inefficiency is the divergence between social value and market value of the patent. But one may either have $V_s > V$ or $V_s < V$, so it is impossible to assess. We suppose $V_s = V$
- *Result 5.* Firms duplicate effort without realizing it. Social optimality thus results in a lower individual level of investment for a fixed n.
- *Result 6.* In the long run, too many firms enter, n is too large with respect to the optimal number of firms in the patent race.

Lee and Wilde 's comment

- In Loury's analysis, firms choose their investment level at the beginning once and for all.
- In Lee and Wilde, firms pay a fixed cost F at the beginning, and then a flow cost x (until one firm has made the discovery)
- The expected cost is then

$$EC = \frac{x}{h(x) + a + r} + F.$$

- The new first order condition is:

$$V = \frac{a + h + r - xh'}{a + rh'}.$$

- Higher values of V result again in higher values of x.

Long run effects in Lee and Wilde

- Equilibrium profit is:

$$E\Pi = \frac{h - xh'}{a + rh'} - F.$$

- Contrary to Loury, $E\Pi > 0$ if returns to scale are increasing ($xh' < h$).
- In addition, we observe that, contrary to Loury, $\frac{\partial x}{\partial a} > 0$: R & D investments are strategic complements.
- This implies that, contrary to Loury, *An increase in competition results in an increase in individual investments*
- Hence the expected time of discovery is decreasing in n.

Monopoly vs competition

- Lee and Wilde compare the optimal investment level of a monopolist running *n* parallel projects and *n* competing firms.
- A monopolist operating the same number of projects will make a total investment in R & D that is less than the aggregate noncooperative investment.
- If the monopolist chooses the number of research teams, he will choose a smaller number of teams than the free entry number of research firms.

Reinganum's model

- Finite time T at which the race is over
- Values of being leader and follower are given by P_L and P_F
- t_i: time at which firm i succeeds
- $\mu_i(t)$: addition accumulated by firm i at date t at the cost $\frac{1}{2}\mu_i(t)^2$
- $z_i(t)$: knowledge accumulated at date t, $z_i(0) = 0$ and $z_i'(t) = \mu_i(t)$
- The probability of discovery is exponential in $z_i(t)$

$$\Pr[t_i \leq t] = 1 - e^{-\lambda z_i(t)}.$$

Strategies and utilities

- Each firm chooses the investment $u_i(t,z)$ as a function of time and the accumulated knowledge z
- The firm obtains a profit P_L if it is the first to obtain the innovation and P_L otherwise,

$$\begin{aligned} J_i &= \int_0^T P_L e^{-\lambda \sum z_k(t)} \lambda \mu_i(t) \\ &+ P_F e^{-\lambda \sum z_k(t)} \sum_{j \neq i} \lambda \mu_j(t) \\ &- e^{-rt} e^{-\lambda \sum z_k(t)} \frac{1}{2} \mu_i(t)^2 dt \end{aligned}$$

Nash equilibrium and value functions

- In this dynamic game, we characterize the Nash equilibrium by solving the Bellman equation with value function:

$$
\begin{aligned}
V_i(s,y) &= \int_s^T (P_L(1-e^{-\lambda z_i(t)}+P_F e^{-\lambda z_i(t)}) \\
&\quad e^{-\lambda \sum_{k\neq i} z_k(t)} \sum_{j\neq i} \lambda\mu^* j \\
&\quad -e^{-rt}e^{-\lambda\sum z_k(t)}\frac{1}{2}\mu_i^*(t)^2 dt \\
&+ P_L(1-e^{-\lambda z_i(T)})e^{\sum_{k\neq i}\lambda z_k(T)}.
\end{aligned}
$$

The maximization problem

- The solution to the maximization problem is given by

$$\begin{aligned} 0 &= V_t^i + \frac{1}{2}(V_{zi}^i)^2 e^{rt} e^{\lambda \sum z_k} \\ &+ \sum_{j \neq i} V_{zj}^i V_{z_j}^j e^{rt} e^{\lambda \sum z_k} \\ &+ (P_L(e^{\lambda z_i} - 1) + P_F)\lambda \sum_{j \neq i} V_{z_j}^j e^{rt}. \end{aligned}$$

- We postulate a solution of the form:

$$V^i(t, z) = b(t) e^{-\lambda z_k} + a(t) e^{-\lambda \sum_{k \neq i} z_k}$$

The differential equations

- Identifying terms in the first order equation, we obtain two differential equations characterizing $a(t)$ and $b(t)$:

$$\begin{aligned} b'(t) + b(t)^2\lambda^2 e^{rt}\frac{2n-1}{2} \\ +b(t)(n-1)(P_L - P_F)\lambda^2 e^{rt} &= 0, \\ a'(t) + a(t)b(t)(n-1)\lambda^2 e^{rt} - b(t)P_L(n-1)\lambda^2 e^{rt} &= 0 \end{aligned}$$

- with the two boundary conditions:

$$\begin{aligned} b(T) &= -P_L, \\ a(T) &= P_L. \end{aligned}$$

The solution to the differential equation

- The solution to the differential equation is

$$b(t) = \frac{-2(n-1)(P_L - P_F)P_L}{(2n-1)P_L - (P_L + 2(n-1)P_F)e^{m(t)}},$$

with

$$m(t) = (P_L - P_F)(n-1)\lambda^2 \frac{(e^{rt} - e^{rT})}{r},$$

and

$$a(t) = P_L.$$

The equilibrium of the dynamic game

Proposition

In Reinganum (1982)'s dynamic patent race, the equilibrium level of investment is given by

$$u_i^* = \frac{2\lambda P_L(P_L - P_F)(n-1)e^{rt}}{(2n-1)P_L - (P_L + 2(n-1)P_F)e^{m(t)}},$$

and the equilibrium payoff by:

$$J^* = \frac{2(P_L - P_F)P_L(n-1)}{(2n-1)P_L - (P_L + 2(n-1)P_F)e^{m(t)}}.$$

Comparative Statics

- As time passes, the instantaneous investment $\mu_i(t)$ increases
- The equilibrium investment is increasing in the value of the leader P_L and decreasing in the value of the follower P_F
- If $P_L > 4P_F(n-1)\frac{e^{-\frac{1}{2}}}{(2n-1-e^{-\frac{1}{2}})}$, equilibrium investment is increasing in the Poisson rate λ. If the condition fails, there exists a time t at which equilibrium investment is decreasing in λ
- Equilibrium investment u^* is decreasing in the time horizon T even though the total accumulated knowledge z^* is increasing in T.

Competition and R & D

- If patent protection is perfect ($P_F = P, P_L = 0$),

$$u^* = \frac{2\lambda(n-1)Pe^{rt}}{2n-1-e^{\lambda^2(n-1)\frac{(e^{rt}-e^{rT})}{r}}},$$

  and $\frac{\partial u^*}{\partial n} > 0$

- It patent protection is imperfect, the result is not clear cut.
- In the limit case where $P_F = P_L$,

$$u^* = \frac{2P_F\lambda e^{rt}}{2-(2n-1)P_F\lambda^2\frac{(e^{rt}-e^{rT})}{r}},$$

  and $\frac{\partial u^*}{\partial n} \leq 0$

Entry

- If firms are free to entry into the patent race at no cost, the number of firms grows to infinity, and each firm invests

$$u^* = P\lambda e^{rt},$$

- This is the competitive level of investment, resulting in the highest probability of innovation.

Asymmetric races

- Firms can have asymmetric values for the patent
- Firms can have different discount factors
- Firms can have different efficiency in research
- Firms can be at different stages of the research process.

A model of race with competitors at different stages

- In Reinganum's model, uncertainties across players are uncorrelated
- A firm's instantaneous probability of success is independent of accumulated knowledge
- *So equilibrium strategies only depend on time, and not on the stage of the race*
- Important questions: If one firm pulls ahead, is the other firm discouraged? Is it trying to catch up?
- To answer those questions, we need a model where a firm's position in the race matters.

The model I

- Players value the patent at V_A, V_B
- Players have discount factors ρ_A and ρ_B
- Players start at a distance x_0, y_0 from the finishing line
- Players bid alternatively A in odd periods, B in even
- The first player to pass the finshing line gets the prize
- By spending z players advance by $w(z)$ with $w(0) = 0$.

The model II

- For player A moving at odd periods

$$\begin{aligned} x_{2k-1} &= x_{2k-2} - w_A(a_k) \\ y_{2k-1} &= y_{2k-2} \end{aligned}$$

- For player B moving at even periods

$$\begin{aligned} x_{2k} &= x_{2k-1} \\ y_{2k} &= y_{2k-1} - w_B(b_k) \end{aligned}$$

- If player A wins in k rounds, her profit is

$$\rho_A^{k-1} V_A - \sum_i \rho_A^{i-1} a_i.$$

The main result

- The space of distances can be partitioned into four regions
- Regions where only A (or B) bid called the ”safety zones”
- Regions where none of them bids
- Regions where the winner is the first to move.

The sequence C_n

- Let $C_0 = 0$.
- C_n is the maximum c for which there exists a sequence of bids $a_1, ..., a_r$ such that
  1. $\sum w_A(a_i) \geq c$
  2. $w_A(a_1) \geq c - C_{n-1}$
  3. $\rho_A^{k-1} V_A - \sum_i \rho_A^{i-1} a_i \geq 0$.
- We define also D_n as a similar sequence for the B player.

The theorem

Proposition

Let x, y be the current distances to the finishing line.

- *If there exists n such that $x \leq C_n$ and $y > D_n$, A is the only player to bid.*
- *If there exists n such that $x > C_n$ and $y \leq D_n$, B is the only player to bid.*
- *If there exists n such that $C_n \leq x, D_n \leq y$, the player who is the first to move is the only one to bid.*
- *If for all n, $x > C_n$ and $y > D_n$, no player bids.*

Conclusions from Harris and Vickers

- There is no race: only one player invests in R & D
- The critical distances are difficult to compute in general.
- When the time between moves goes to zero, let r_A,r_B be the discount factors in continuous time and $w(z) = z^\eta$
- The safety zones od A and B are separated by the line:

$$y = (\frac{v_B}{V_A})^\eta(\frac{r_A}{r_B})^{1-\eta}x.$$

Fudenberg, Gilbert, Stiglitz, Tirole (1983)

- The paper puts together results independently found by Gilbert and Stiglitz and Fudenberg and Tirole on the same ideas.
- As in Harris and Vickers, the firm's position in the patent race affects its success probability
- The model is simpler than Harris Vickers (only two steps, fixed research intensity) but very insightful.

A model with fixed research intensities

- Suppose that the stock of knowledge of firm i depends on the number of periods it has engaged in R & D,

$$w_i(t) = \int_{t_i}^{t} e_i(\tau)d\tau,$$

where $e_i(\tau) = 1$ if the firm engages in R& D at period τ.

- Suppose that the instantaneous probability of success is increasing in the stock of knowledge, $\mu_i(w_i(t))$
- The expected profit of research is

$$\Pi_i = \int_{t_i}^{\infty} e^{-rt+\int_0^t(\mu_1(\tau)+\mu_2(\tau))} [\mu_i(t)V - c]dt$$

Preemption in the patent race

Proposition

Suppose that research is profitable for a monopolist but not for duopolists and that firm 1 *starts research at* $t_1 < t_2$. *Then, in equilibrium, only firm* 1 *invests in R & D.*

A two stage patent race

- Suppose that there are two successive discoveries to be made.
- Each discovery is independent of the other (knowledge accumulated in stage 1 cannot be used in stage 2
- Stage 1 has to be completed before stage 2
- Firm 2 even if it trails in stage 1 can leapfrog by accomplishing stage 1 before firm 1.
- the incentives to preempt and leapfrog are not clear anymore.

An example with both leapfrogging and preemption

Proposition

Suppose that the instantaneous probability of success is increasing in knowledge in the first stage, constant in the second. The leading firm always does R & D except if the follower completes the first stage before a fixed date ω_1. *The follower, depending on the parameters, either drops from the race, or invests until time* ω_1 *or always invests if the leader does not find the first stage before* $\omega_1 + t_2$ *where* t_2 *is the headstart of firm* 1 *in the race.*

A deterministic patent race

- In the last model, Fudenberg et al (1983) consider a dterministic patent race as Harris and Vickers
- First must complete N steps to obtain the patent
- At each time period they have the choice between two units of effort (yielding advance 1 or 2) at costs $c_1 < c_2$
- Let $\overline{k}$ be the largest integer such that $\frac{V}{2} - c_2\frac{k}{2} > 0$.
- Suppose $c_2 > 2c_1$, $\frac{V}{2} - c_2 \geq 0$ and $N \leq 2\overline{k}$.

Equilibrium of the deterministic patent race

Proposition

If the follower is behind with two or more steps, it drops out. If it is one step behind and the number of remaining steps is not too large, it randomizes between the high effort and zero, and the leader randomizes between low and high effort. If the two firms are tied they choose the high cost if the number of remaining steps is below $\overline{k} + 1$ *and randomize between the high and low efforts before.*