Industry dynamics

Francis Bloch

January 7, 2014

Industry dynamics

- We now consider the level of the *industry*
- What are the industry dynamics? Which firms grow, decline?
- How can one explain the size distribution of firms in an industry?
- How can one explain exit and entry rates in an industry?

Empirical evidence

- Empirical evidence is mixed:
    - Early models suggest that firms' growth is proportional to their size.
    - Hence large firms grow, small firms decline, and the industry becomes more and more concentrated.
    - More recent models suggest that small firms have a higher mean growth rate but with more variance
    - Hence, small firms exit more frequently, but may grow faster than large firms.
- The challenge is to build a model of industry dynamics capturing the main stylized facts, and test it with empirical data.

Three models

- Lucas (Bell Journal, 1978): size distribution of firms
- Jovanovic (Econometrica, 1982): selection in an industry through costs
- Hopenhayn (Econometrica 1992): entry and exit dynamics
- Ercison and Pakes (1995): industry dynamics with empirical application

Old theories of size distribution

- Jacob Viner (1932): unique size within an industry if all firms have U shaped long run average cost functions.
- The evidence against Viner's one firm size is overwhelming (except for store or plant size)
- Most changes are due to adjustment of output within firms, not entry exit
- The growth rate is not directly proportional to size.
- Manne (1965): size distribution is a solution to the problem: allocate productive factors to managers of different abilities to maximize output.

The model of allocation of productive units

- A workforce of size N and K units of capital.
- A constant returns to scale production technology $Y = f(n, k) = n\phi(r)$ where $r = \frac{k}{n}$.
- If everyone in the economy had perfect management skills, output would be $Y = N\phi(R)$, with $R = \frac{K}{N}$, wage $w = \phi(R) - R\phi'(R)$ and capital rent $\phi'(R)$.

Managerial skills

- Managerial skills involve idiosyncratic talent x drawn from a distribution Γ and diminishing returns to scale,

$$y = xg[f(n,k)],$$

where $g(\cdot)$ is increasing and strictly concave with $g(0) = 0$.

- There is a continuum of agents. An allocation is a pair of functions $n(x), k(x)$ assigning employees and capital to a manager of ability x.
- If $x < z$, the worker is an employee, if $x \geq z$, a manager.

Feasible and efficient allocations

- An allocation is *feasible* (in terms of labor) if

$$1 - \Gamma(z) + \int_z^1 n(x) d\Gamma(x) \leq 1.$$

- in terms of capital

$$\int_z^1 k(x) d\Gamma(x) \leq \frac{K}{N} = R.$$

- An allocation is *efficient* if it maximizes

$$\frac{Y}{N} = \int_z^1 xg[f(n(x), k(x)] d\Gamma(x).$$

The efficient allocation

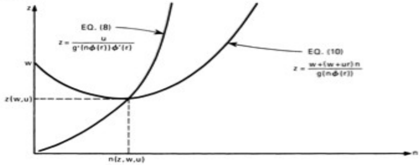
- Let w and u be the Lagrange multipliers of the two constraints
- The planner chooses $n(x)$ and $k(x)$ for all x and the threshold value z.
- the first order conditions (using constant returns to scale) are

$$xg'[n(x)\phi(r)]\phi'(r) = u,$$

$$zg[n(z)\phi(r)] = w + (w + ur)n(z).$$

Equilibrium (n, z) in Lucas's model

FIGURE 1



Dynamic adjustment costs

- The model is static and has no implications for growth.
- Imagine instead that mangers reallocate resources over time and consider

$$\frac{d}{dt}\ln[n(x, w(t), u(t)]$$

$$\frac{d}{dt}\ln[r(w(t), u(t)0n[x(w(t), u(t)]]$$

- Gibrat law states that the derivatives of growth rates with respect to x are equal to zero,

$$0 = \frac{\partial}{\partial x}\frac{n_w(x, w, u)}{n(x, w, u)}\frac{dw}{dt} + \frac{\partial}{\partial x}\frac{n_u(x, w, u)}{n(x, w, u)}\frac{du}{dt}$$

Gibrat law

- So that

$$\frac{\partial}{\partial x}\frac{n_w(x,w,u)}{n(x,w,u)} = \frac{\partial}{\partial x}\frac{n_u(x,w,u)}{n(x,w,u)} = 0.$$

- resolving

$$n_w(x,w,u) = -r_w \frac{w}{u}\frac{g'(f)\phi''(r) + g''(f)\phi'(r)^2 n}{g''(f)\phi(r)\phi'(r)}.$$

- yielding $g(v) = \alpha v^{\beta}$.
- and we compute

$$n(x,w,u) = \frac{1}{\phi(r)}[\frac{u}{\alpha\beta x\phi'(r)}]^{\frac{1}{\beta-1}}.$$

- and z is given by

$$0 = \frac{\beta}{\Gamma}[1 - \frac{r\phi'(r)}{\phi} - (1-\beta)z^{\frac{1}{1-\beta}}].$$

Comparative statics

- In the Cobb Douglas case, the cutoff point z does not depend on R.
- Average firm size is increasing with the per capita wealth, R.
- An increase in R raises wages relative to managerial rents, making the number of entrepreneurs go down and increasing the average size of firms.
- For a fixed x, we can compute the number of employees as the solution to

$$x = [\frac{L(z)}{\Gamma(z)}]^{1-\beta} n^{1-\beta}.$$

The Pareto distribution example

- Let $\Gamma(x)$ be a Pareto distribution

$$\Gamma(x) = 1 - B^{\rho} x^{-\rho}.$$

- Then the distribution of firms with less than n employees is also a Pareto distribution

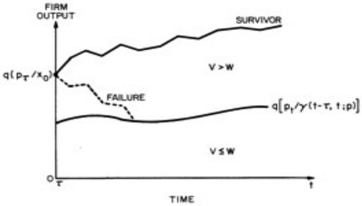
$$S(n) = 1 - z^{\rho}[\frac{L(z)}{\Gamma(z)}]^{-\rho(1-\beta)} n^{-\rho(1-\beta)}.$$

- The rent to managers is proportional to $n(x)$,
- Average firm size is an increasing function of z. This value increases with the capital labor ratio R.

The main idea

- A small industry where factors are supplied at constant price
- Demand is deterministic and known
- Firms have unknown costs, and selection operates through random shocks on costs
- More efficient firms grow and survive
- Less efficient firms decline and exit.

Selection in Jovanovic's model



Firms costs

- A continuum of firms, each with a cost function $c(q_t)$ where
  - $c(0) = 0, 4c'(0) = 0, c'(q) > 0, c''(q) > 0$ and $\lim_{q \to \infty} c'(q) = \infty$.
  - Total costs are $c(q_t)x_t$ where x_t is a random variable independent across firms.
  - For a firm of type θ, $x_t = \xi(\eta_t)$ where $\xi(\cdot)$ is, positive, strictly increasing, continuous with
    - $\lim_{\eta_t \to -\infty} \xi(\eta_t) = \alpha_1 > 0$ and $\lim_{\eta_t \to \infty} \xi(\eta_t) = \alpha_2$
    - $\eta_t = \theta + \epsilon_t$, $\epsilon_t \sim \mathcal{N}(0, \sigma^2)$

Profits

- Among potential firms, θ is normally distributed with mean $\overline{\theta}$ and variance σ_θ^2
- An entrant does not know his θ but knows that it is a random draw from the Normal distribution.
- The firm takes the price as given and chooses q_t to maximize

$$p_t q_t - c(q_t) x_t^*,$$

where x_t^* is the expectation of x_t conditional on information received until t.

Output decision

- The output decision is made before x_t is known and satisfies

$$\frac{\partial q}{\partial x_t^*} = -\frac{c'}{x_t^* c''} < 0.$$

$$\frac{\partial^2 q}{\partial x_t^{*2}} = \frac{1}{x_t^*}\left[\frac{c' c'''}{(c'')^2} - 2\right]\frac{\partial q}{\partial x_t^*}.$$

The exit/entry decision

- Let $W > 0$ be the reservation value of the firm if assets are employed in a different activity.
- The value of W is the same for all firms, uncorrelated with θ
- A cost of entry k s borne when he firm enters.
- Firms have a discount rate r.
- As time passes, firms learn about θ.
- At time t, it has a pair of sufficient statistics $(\overline{\eta_n}, n)$ where n is the number of periods the firm has been active and $\overline{\eta}_n$ the mean value of η.

Value of the firm

- Decisions are not stationary because the deterministic path price $\{p_0, ..p_t, ..\}$ is not constant
- The value of the problem is

$$\begin{aligned} V(x,n,t;p) &= \pi(p_t,x) \\ &+ \beta \max\{W, V(z,n+1,t+1;p)\}P(dz|x,n) \end{aligned}$$

Proposition

A unique, bounded and continuous function for V exists and V is strictly decreasing in x.

Exit decision

- Let $\gamma(n, t; p)$ be the value of x at which the firm stops, i.e.

$$V(x, n, t; p) = W.$$

- The output level below which the firm exits is $q(\frac{p_t}{\gamma(n,t;p)})$.
- We let τ define the time at which the firm enters the industry. When it enters and has prior information $x = x_0$ and a net value $V(x_0, 0, t; p) - k$
- As V is decreasing in x, γ is uniquely defined.
- This boundary defines an "exit region" where $V \leq W$ and a continuation region where $V > W$.

Time paths

- The firm's output sequence $q(\frac{p_t}{x_t^*})$ is a random process starting with $q(\frac{p_T}{x_0})$.
- The x_t^* sequence is a martingale, $Ex_{t+k}^* = x_t^*$ for any $k > 0$.
- If output remains in the good area, the firm survives.
- The smaller firms are more likely to fail.
- The x_t^* sequences are different among firms, and tend to diverge. So do outputs.

Industry dynamics

- The Gini coefficient of firm sizes increases over time (as observed in UK data)
- If the equilibrium price is constant, selection occurs over time, so the average profit rises in the industry.
- There is thus a positive correlation between profits and concentration.
- If the price sequence falls over time, this may offset the selection effect, and profits may go down.
- Higher concentration increases the profit of large but not small firms because small firms do not earn rents.

Industry dynamics

- There is a positive relation between variability of profits and concentration. If $\sigma_t^2 = 0$, variability of profit is zero and so is the Gini coefficient.
- Both output and profits are correlated with x so there is a one to one relation between inequality of firm size and variability of profits.
- Unusually high profits today result in unusually high growth between today and tomorrow. the firm's revision of x_t^* depends on realized profit

$$\pi_t - E_t \Pi_t = -c(q_t)(x_t - x_t^*).$$

- If profits are high, x_t is revised downwards, and next period output will be larger than usual.

Industry dynamics

- Fluctuations in output occur in every industry.
- Which fraction is due to fluctuations in the output of existing firms, and which in entry and exit?
- If $q(\cdot)$ is concave, and p constant, $E_t q(\frac{p}{x^*_{t+1}}) < q(\frac{p}{x_t})$. Output of existing firms goes down every period, any increase in output is due tot the entry of new firms.
- Equilibrium always coincides with the maximization of discounted consumer surplus (no externalities, competitive markets)
- Entry and exit occur neither too early nor too late.

Growth rates

- The variability of growth rates is highest among the young and smaller firms.
- For large surviving firms, x_t^* converges to a constant.
- Hence we must have

$$\frac{1}{q}\frac{dq}{dp_t} = k(p_t),$$

where $k(p_t)$ does not depend on firm size and hence not on x.

Growth rates

- Solving the differential equation

$$q(\frac{p_t}{x}) = \delta_1 [\frac{p_t}{x}]^{\delta}.$$

- where δ_1 and δ are positive constants.
- This can only happen if $c(q)$ assumes the Cobb Douglas form

$$c(q) = \beta_1 q^{\beta_2}.$$

- and the growth rate is then

$$[\frac{p_{t+1}}{p_t}]^{\delta} [\frac{x_t^*}{x_{t+1}^*}]^{\delta} - 1.$$

Growth rates

- Let $z_t \equiv \frac{x_t^*}{x_{t+1}^*}$.
- Weak proportionality: Ez_t^δ is the same for all x_t^*.
- Strong proportionality: The entire distribution of z_t is the same for all firms.
- The strong proportionality has been empirically rejected and cannot happen in the model: two firms with the same x_t^* but different precisions will have different z_t.
- But z_t can have the same distribution for two firms of the same cohort.
- However, even the weak form of proportionality does not occur in the model.

Hopenhayn (1992): Stylized facts

- Firm specific uncertainty plays a large role in firm size dynamics
- A third of the stock of jobs and forty percent of the firms in manufacturing disappear over five year periods and are replaced by new ones.
- Entry and exit rates are highly correlated across industries, and most of their variation is accounted for by these industry effects.
- High and low turnover industries.

The model

- Firms are faced with individual productivity shocks
- On the basis of these shocks, they decide when to exit.
- Entry requires a sunk investment
- In the steady state, entry and exit rates are equal, the distributions of profits, firm size and value are invariant.
- This paper discusses the comparative statics at the steady state.

Relation to the literature

- The models of Jovanovic (1982) and Ericson and Pakes (1995) deal with the same issue, but are more complex.
- They give results at the firm level, but not at the aggregate industry level which is too complex.
- In Hopenhayn (1992), the structure is simpler and attention focused on a *stationary equilibrium*

The model

- A continuum of firms producing a homogeneous product
- Firms are competitive, aggregate demand is given by $D(Q)$ and input price $W(N)$ where N is total industry demand for the input (labor)
- *Assumption* D is continuous, strictly decreasing with $\lim_{x \to \infty} D(x) = 0$, W is continuous, nondecreasing and strictly bounded above zero.
- The output of an individual firm is $q = f(\phi, n)$ where $\phi \in [0, 1]$ is a productivity shock which follows a Markov process independent across firms with conditional distribution $F(\phi' | \phi)$

The model

- A fixed cost c_f must be paid every period by incumbent firms.
- For given output and input prices p, w, $\pi(\phi, p, w), q(\phi, p, w)$ and $n(\phi, p, w)$ are profit, output and input demand functions.
- Firms discount profit with the discount factor β.
- *Assumption.* q and n are single valued, strictly increasing in ϕ and continuous. π is continuous strictly increasing in ϕ, $\lim_{Q\to 0} \pi(0, D(Q), w) > 0$.
- *Assumption.* F is continuous in ϕ and ϕ', F is strictly decreasing in ϕ.
- *Assumption.* For any $\epsilon > 0$ there exists an integer n such that $F^n(\epsilon|\phi) > 0$

The model

- Higher productivity shocks make high productivity more likely
- In equilibrium, firms exit when the shock falls below a level x. Given this cutoff point, the lifespan of a firm is almost surely finite.
- When a firm exits, its present value is zero. When a firm enters it pays a sunk cost c_e
- μ is the distribution of productivity shocks at any time t.

Aggregate output supply and input demand functions

$$
\begin{aligned}
Q^s(\mu, p, w) &= \int q(\phi, p, w)\mu(d\phi), \\
N^d(\mu, p, w) &= \int n(\phi, p, w)\mu(d\phi)
\end{aligned}
$$

- The sequence of output and input prices $z = \{p_t, w_t\}$ is deterministic and known.

Exit

- Given the price sequence, the problem of an incumbent is

$$v_t(\phi, z) = \max \pi(\phi, p_t, w_t) + \beta \max\{0, \int v_{t+1}(\phi', z) F(d\phi'|\phi)\}.$$

- involving a reservation rule

$$x_t = \phi | \int v_{t+1}(\phi', z) F(d\phi'|\phi) = 0.$$

- The firm exits the industry the first time the shock hits below the reservation value.

Entry

- New firms enter until the expected discounted profits net of the entry cost equals zero

$$v_t^e(z) = \int v_t(\phi, z)\nu(d\phi) = c_e.$$

- The mass of entrants is M_t
- The evolution of the state of the industry is thus

$$\mu_{t+1} = \int_{\phi \geq x_t} F(\phi'|\phi)\mu_t(d\phi) + M_{t+1}G(\phi').$$

Competitive equilibrium

- A competitive equilibrium consists of $p_t^*, w_t^*, Q_t^*, N_t^*, M_t^*, \mu_t^*$ such that
  - $p_t^* = D(Q_t^*), w_t^* = W(N_t^*)$
  - $Q_t^* = Q^s(\mu_t^*, p_t^*, w_t^*), N_t^* = N^d(\mu_t^*, p_t^*, w_t^*)$
  - x_t^* satisfies the stopping rule
  - $V_t^e(z^*) \leq c_e$ with equality if $M_t^* > 0$
  - μ_t^* is obtained recursively by using the law of motion.

Existence and uniqueness of equilibrium

Proposition

Given any initial distribution, there exists a unique equilibrium.

Proposition

The unique equilibrium maximizes social surplus.

Stationary equilibria

- An equilibrium is stationary if $p_t = p^*, w_t = w^*, Q_t = Q_t^*, N_t = N^*, x_t = x^*, M_t = M^*, \mu_t = \mu^*$
- Given the distribution of firms μ, there exist unique imput-ouput vector and prices resulting in equilibrium.
- We define the equilibrium quantities and price as $Q^e(\mu), N^e(\mu), p^e(\mu), w^e(\mu)$
- Define $v(\mu, \phi)$as the unique solution to

$$v(\phi, \mu) = \tilde{\pi}(\phi, \mu) + \max\{0, \beta \int v(\phi', \mu)\} F(d\phi'|\phi).$$

- There is a unique solution v increasing in ϕ and decreasing in μ.

Existence and uniqueness of stationary equilibrium

- We need to show existence of the threshold x, mass of entrants M and distribution of firms μ satisfying equilibrium conditions.
- This is done using a fixed point argument.

Proposition

There exists a stationary competitive equilibrium for the industry.

Proposition

There is a real number c^* such that a stationary equilibrium with positive entry exists if and only if $c_e \leq c^*$.

Birth, growth and death of firms

- The evolution of the industry is governed by the initial distribution ν of entering firms, and the Markov process F
- An empirical regularity is that size distribution is stochastically increasing with age. Do we get it here?

Proposition

The distribution μ_t always stochastically dominates the distribution ν

- This does not prove that the distribution μ_t is increasing over time.
- Under strict conditions on F, μ_{t+1} dominates stochastically μ_t

Birth, growth and death of firms

- The distribution of firms' shocks is increasing in the age of the cohort, the rate of survival is higher for older firms, as well as average size, profits and values of firms.
- Empirical studies show some regression to the mean in size but higher persistence for larger and older firms.
- The model is consistent with these observations for appropriately chosen F.

Comparative statics: entry cost

- An increase in c_e shifts x^* downwards and M^* too.
- There is less selection and higher expected lifetime of entry. The cost of entry acts as a barrier to entry, protecting incumbent firms.
- The effect of an increase in c_e on size distribution is not obvious.
  - There is a price effect and a selection effect.
  - Output prices increase because of the barrier to entry resulting in larger firms
  - But because x^* goes down, smaller firms also are more likely to survive.

Comparative statics: distribution of entrants

- Suppose that ν stochastically decreases.
- The direct effect is that entrants have lower profits
- The indirect effect is that the invariant distribution puts more weight on small firms, increasing the profit of entrants.
- What if we take a mean preserving spread of ν?
- Because the value function is convex in ϕ, this results in higher expected profits for the entrants.
- More variability results in larger firms

Comparative statics: fixed cost of production

- Suppose that the fixed cost of production, incurred every period, increases.
- An increase in the fixed cost raises x^* making exit more likely
- This implies that larger firms are more likely to survive when c_f goes up.

Profits and values of firms

- Let $\rho = \frac{M^*}{\mu^*(S)}$ be the *turnover rate*

Proposition

At the stationary equilibrium,

$$\overline{\pi(\mu^*)} = \int \pi(\phi, \mu^*)\mu^*(d\phi) \geq \rho c_e.$$

Proposition

At the stationary equilibrium,

$$\overline{v(\mu^*)} = \in \overline{\pi}(\mu^*) - \frac{\beta \rho c_e}{1 - \beta} \geq \rho c_e.$$

Hopenhayn's model: conclusions

- Entry and exit happen in the stationary distribution
- (In other papers, like Jovanovic, in the long run, only some firms survive)
- The only decision of the firm is the stopping decision , as in Jovanovic
- We could add more variables for firms: investments, allow them to modify the distribution of shocks..

Overview of the model

- There are profit opportunities that firms must exploit.
- In order to exploit them, firms must invest in a sunk cost. The quantity of investment determines the distribution of outcomes every period. (different from Jovanovic and Hopenhayn)
- Favorable outcomes moves the firms to better state ; favorable outcomes of direct competitors tend to move the firm to less favorable states.
- The opportunity is open to all, the only distinction among firms is their state of success ω taken to belong to the set of positive integers.

The model

- The state ω changes over time as a result of autonomous factors which shift demand, cost parameters
- Higher ω means that the firm is in a stronger position.
- The sequence s_ω determines the distribution of firms in state ω
- A *state* in the model is a pair (s, ω)
- The equilibrium determines a ranking over s, indicating for a fixed firm of state ω, which of two distributions s and s' lead to a better competitive outcome.
- Current profits are weakly decreasing in s for fixed ω
- Current profits are weakly increasing in ω for fixed s.

Evolution

- The state (ω, s) changes as the outcome of the firm's own investment, other firms' investments and changes in the market environment.
- The firm's investment x_t maximizes expected discounted profit given the information at date t
- Outcomes in the exogenous stochastic process generate a correlated shift in all the firms' ωs
- A new entrant incurs a sunk entry cost x^e and then takes up one period to set up the capital. the sate of an entrant is drawn from a distribution $P(\omega^0)$

Primitives of the model

- $A(\omega, s)$: profit of the firm given ω, s
- $p(\omega'|\omega, s)$: probability of shift in the firm's type
- $q_\omega(s'|s)$: belief on the transition probabilities of other firms
- $m(s)$: number of entrants stimulated by s
- $P(\omega^0)$: probability of type at entry
- x^e_m, $m = 1, ..\infty$: initial investment needed to enter

Primitives of the model

- ϕ: opportunity cost of being in the industry
- $c(\omega)$: unit cost of activity level x
- Profits are $R(\omega, s, x) = A(\omega, s) - c(\omega)x$.
- β is the common discount factor

Assumptions

- $A(\omega, s)$ is increasing in ω, decreasing in s
- The transition p is given by $\omega' = \omega + \tau + \eta$, where τ is determined by the firm's investment and η the process defining the firm's outside alternative
- $m(s)$ firms enter each period . Each entrant pays $x_m^e > \beta\phi$, increasing in m.

The incumbent's decision

- The incumbent decides whether to stay or exit and if it stays, how much to invest.

$$W_t(\omega_t, s_t) = \max_{x_t, \chi_t} E_t[\sum_{\tau=t}^{\infty} \beta^{\tau-1} R(\omega_\tau, s_\tau, x_\tau)\chi_\tau + (\chi_\tau - \chi_{\tau-1})\phi,]$$

  where χ_t is the continuation decision ($\chi = 1$ means continue, $\chi = 0$ stop.)

- The optimal strategy is a Markovian strategy $(x(\omega, s), \chi(\omega, s))$ with the Bellman equation

$$V(\omega, s) = \max_x \{R(\omega, s, x) + \beta EV(\omega', s' | \omega, s)\}$$

The entrant's decision

- The entrant firm decides whether or not to pay the entry cost x^e_m anticipating the number of other entrants m, its possible type ω^0 and the state s_{t+1} of next period, when it will start to produce and become an incumbent.

$$V^e(s, m) = \beta EV(\omega^0, s_{t+1}).$$

- If $V^e(s, m) \leq x^e_m$, for all $m \geq 1$ no entry takes place
- If $V^e(s, m+1) - x^e_{m+1} \leq 0 < V^e(s, m) - x^e_m$, m firms enter.

Equilibrium

- We consider a rational expectations equilibrium where
  - Incumbent firms choose optimally x, χ every period anticipating a stochastic path of (ω, s)
  - Entrants choose optimally whether to enter every period anticipating a stochastic path (ω, p)
  - The stochastic path (ω, p) is generated by the distributions p and q given the incumbent and entrant's decisions.

Value functions

Proposition

- *There exists a unique Markovian value function* $V(\omega, s)$ *monotonic in* ω, *a unique optimal investment policy* $x(\omega, s)$ *and a unique termination policy* $\chi(\omega, s)$.
- *There exist two boundaries* $\underline{\omega(s)}$ *and* $\overline{\omega(s)}$ *such that* $x(\omega, s) = 0$ *if* $\omega < \underline{\omega(s)}$ *or* $\omega > \overline{\omega(s)}$.
- *There exists* $\underline{\underline{\omega(s)}}$ *such that* $\chi(\omega, s) = 0$ *if* $\omega < \underline{\underline{\omega(s)}}$.
- *There exists a random variable* $T(\omega, s)$ *associating to each* (ω, s) *with the first time at which the firm exits.*

Characterization of the incumbent's behavior

- The incumbent's behavior is well defined
- In good states, the incumbent does not invest because the expected marginal gain to advance is too low
- In bad states, the incumbent does not invest because the profit is too low. It may be that the incumbent stays ($\chi = 1$) but chooses $x = 0$.
- If states are too low, the incumbent prefers to liquidate.

Selection in the Ericson Pakes model

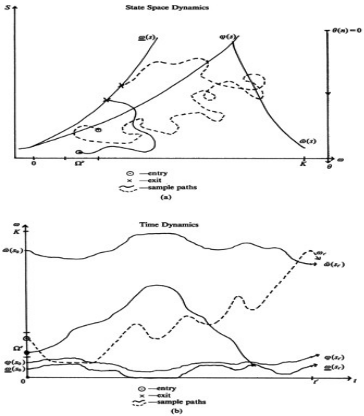


FIGURE 1

Existence of equilibrium, entry

Proposition

A rational expectations equilibrium exists.

Proposition

There exists M such that for all $m \geq M$, $V^e(s, m) \leq x^e_m$ (entry is bounded)

Equilibrium dynamics

- Active firms are heterogeneous in ω
- Multiple rank reversals are possible in sales, profitability, employment over the lifetime.
- The structure of the industry can change drastically over time, even though it remains finite.
- What is the long run behavior of the process?

Ergodic dyamics

Proposition

The stochastic process generating the distribution of firms s is ergodic: for any initial condition, there exists a unique recurrent class $R \subset S$, with invariant probability measure μ^*.

- The industry structure evolves in a non degenerate but increasingly regular way.
- There is no limit structure of equilibrium, because all structures in R are realized with positive probability.
- After some time, a certain regularity will appear
- The influence of any initial condition will fade away.
- In Jovanovic, there is no entry and exit in the limit.
- Hopenhayn (1982)'s model is ergodic but shocks are exogenous.

The long run dynamics

- We don't know the characteristics of the ergodic distribution: large number of small firms, or small number of large firms?
- Are the states in the recurrence class similar so that the industry settles down after a finite number of periods?
- Or does this class contain very different states?
- How long will it take for the process to enter the recurrence class?

An Example

- Producers with different constant marginal costs θ_ω, determined by a firm specific efficiency index τ and a common factor price index η, $\omega = \tau + \eta$ and $\theta_\omega = \gamma e^{-\omega}$.
- Firms' investment in R and D increases τ, η is a nondecreasing stochastic process generating the correlated negative drift in the industry.
- The spot market equilibrium is Nash in quantities.
- Demand is $D - Q$,
- Profits are
$$A_i = (D - Q)q_i - \theta_i q_i - f.$$
- Transitions are given by $\pi(\tau = 1|x) = \frac{ax}{1+ax}$, $p(\eta = 0) = 1 - \delta$

Simulations

- Using numerical methods, we compute the long run behavior of the model for the Markov Perfect Nash equilibrium, the collusive equilibrium, the planner.
- We calibrate te model at $D = 4, f = 0.2, x^e = 0.4, \phi = 0.2, c = 1, \beta = 0.925$.

Simulation results

Percent of periods	MPN	Collusion	Planner
1 firm	27.9	92.4	98.3
2 firms	70.8	7.6	1.7
3 firms	1.2	0	0
Entry + Exit	16.5	5.4	1.2
Entry or exit	20.4	10.0	2.1

Simulation results

Averages	MPN	Collusion	Planner
Price	1.79	2.22	0.15
Entry	0.19	0.08	0.02
Active firms	1.74	1.08	1.02
Welfare	36.5	28.1	58.8

Optimal Investment in the Ericson Pakes model

Optimal Investment: Firm 1

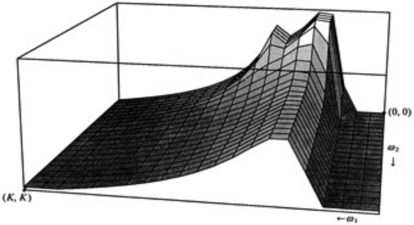


FIGURE 2