Adoption, Entry and Exit

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Adoption, entry and exit

- We consider games where firms make an irreversible decision at some point in time t
  - Adopt a new technology / Enter a market (Reinganum, 1981) or (Fudenberg and Tirole, 1985)
  - Exit a market (Fudenberg, Tirole, 1986)
- There is a dynamic element (change in the environment either due to learning a parameter or a learning curve effect)
- Close to models of stopping or quitting games in the game theory OR literature
- Related to models of investment and real options (book by Dixit and Pyndick ”Investment under uncertainty”, 1994)

Relevance

- Models of entry and exit are important to understand the evolution of an industry
- They allow to study dynamic trajectories of prices, investments and quantities produced
- They rely on ideas of *preemption*: firms want to preempt their competitors or *attrition*: firms want to stay longer than their rivals.

Reinganum (1981)'s model of adoption

- Suppose that a cost reducing innovation has been discovered.
- How and when will the firms adopt the new technology?
- Firms benefit from the new technology and have an incentive to preempt their rival (strategic incentive to invest early)
- Firms prefer to wait because the cost of introducing the new technology goes down
- How doe those two incentives (to adopt early and to wait) play out?

The model

- Two firms.
- Before adoption of technology profits are (π_0, π_0)
- If firm 1 innovates not 2, profits are (π_1, π_2)
- After both firms innovate, profits are (π_3, π_3)
- If a firm adopts at time t, the price of the innovation is $p(t)$.

Profits of the two firms

t	1	2
$0 \leq t \leq \min\{T_1, T_2\}$	π_0	π_0
$T_1 \leq t \leq T_2$	π_1	π_2
$T_2 \leq t \leq T_1$	π_2	π_1
$t \geq \max\{T_1, T_2\}$	π_3	π_3

Assumptions

- **A1** $\pi_i > 0$
- **A2** $\pi_1 > \pi_3 > \pi_2, \pi_1 > \pi_0 > \pi_2$
- **A3** $\pi_1 - \pi_0 + \pi_2 - \pi_3 > 0$
- **A4** $p(t) \geq 0, p'(t) \leq 0, p''(t) \geq r(\pi_1 - \pi_0)e^{-rt}$
- **A5a** $p'(0) < \pi_2 - \pi_3$
- **A5b** $p'(0) \geq \pi_2 - \pi_3$

Values of the firms

- Let T_1, T_2 be the adoption dates of the firms,
- $V_1(T_1, T_2) = g^1(T_1, T_2)$ if $T_1 \leq T_2$
- $V_1(T_1, T_2) = g^2(T_1, T_2)$ if $T_1 \geq T_2$

$$
\begin{aligned}
g^1(T_1, T_2) &= \int_0^{T_1} \pi_0 e^{-rt} dt + \int_{T_1}^{T_2} \pi_1 e^{-rt} dt \\
&+ \int_{T_2}^{\infty} \pi_3 e^{-rt} dt - p(T_1), \\
g^2(T_1, T_2) &= \int_0^{T_2} \pi_0 e^{-rt} dt + \int_{T_2}^{T_1} \pi_2 e^{-rt} dt \\
&+ \int_{T_1}^{\infty} \pi_3 e^{-rt} dt - p(T_1)
\end{aligned}
$$

Optimal adoption times

Proposition

There exists a unique $\hat{T} \in [0, \infty)$ *and* $\hat{\hat{T}} \in [0, \infty)$ *which maximize* $g^1(T_1, T_2)$ *and* $g^2(T_1, T_2)$ *independent of* T_2*.*

- *If A5a holds,* $0 \leq \hat{T} < \infty$, $0 < \hat{\hat{T}} < \infty$
- *If A5b holds,* $\hat{T} = \hat{\hat{T}} = 0$.

- If A5a holds, $\hat{T}$ is defined by $(\pi_0 - \pi_1)e^{-rt} - p'(t) = 0$
- If A5a holds, $\hat{\hat{T}}$ is defined by $(\pi_3 - \pi_2)e^{-rt} - p'(t) = 0$
- We conclude $\hat{T} < \hat{\hat{T}}$

Main result

Proposition

If Assumption A5a holds, there are two Nash equilibria of the game in pure strategies, $(T_1, T_2) = (\hat{T}, \hat{\hat{T}})$, $(T_1, T_2) = (\hat{\hat{T}}, \hat{T})$. *If Assumption A5b holds, there is a unique equilibrium in pure strategies,* $(T_1, T_2) = (0, 0)$.

Interpretation

- the two firms adopt the innovation at different times: there is not immediate diffusion of new technologies
- An increase in $\pi_1 - \pi_0$ (respectively $\pi_3 - \pi_2$) reduces the time of preemption $\hat{T}$ (respectively $\hat{\hat{T}}$)
- Compared to a monopolist, one of the firm invests too early, the other one too late
- Reinganum looks at an "open loop' Nash equilibrium. Firms choose their adoption times at the beginning of the dynamic game.

Entry game in Fudenberg and Tirole

- In Reinganum (1981), firms precommit to an adoption time
- Fudenberg and Tirole (1985) look at a dynamic game (in continuous time) where firms condition their adoption decision at period t on what they have observed in the game
- This is a ”closed loop equilibrium” rather than an ”open loop equilibrium” (this distinction is similar to the distinction between Nash and sub game perfect equilibrium)
- Fudenberg and Tirole obtain a very different result than Reinganum.

The model

- n firms
- At date 0 an innovation is announced, and the cost is $c(t)$
- Firms get flow payoffs $\pi_0(m)$ and $\pi_1(m)$ if they do not (or adopt) the innovation and m other firms have already adopted.
- Time is continuous, $t \in (0, \infty)$ and firms choose a strategy which is a probability to adopt at or before any time t.

The value of firms

- A firm's value is

$$\begin{aligned} V_i(T_i, T_{-i}) &= \sum_{m=0}^{i-1} \int_{T_m}^{T_{m+1}} \pi_0(m) e^{-rt} dt \\ &+ \sum_{m=i}^{n} \int_{T_m}^{T_{m+1}} \pi_1(m) e^{-rt} dt - c(T_i) \end{aligned}$$

Assumptions

- For all m, $\pi_1(m) > \pi_0(m)$
- For all m, $\pi_i(m) < \pi_i(m-1)$
- For all m, $\pi_1(m) - \pi_0(m) < \pi_1(m-1) - \pi_0(m-1)$
- $\pi_1(1) - \pi_0(n-1) \leq -c'(0)$
- $\min_t c(t)e^{rt} < \frac{\pi_1(n) - \pi_0(n-1)}{r}$
- $(c(t)e^{rt})' < 0, (c(t)e^{rt})'' > 0.$

Precommitment equilibrium

Proposition

In the recommitment equilibrium (cf Reinganum), firms which adopt earlier have a higher payoff

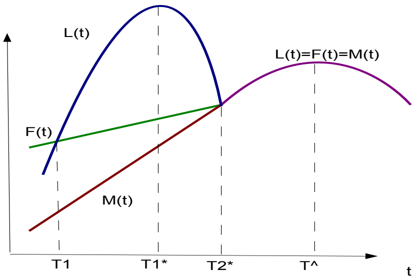
- This result suggests that firms have an incentive to preempt and be the first to innovate.
- The optimal adoption time of the m th firm is defined implicitly by

$$(\pi_1(m) - \pi_0(m-1))e^{-rT_m^*} + c'(T_m^*) = 0.$$

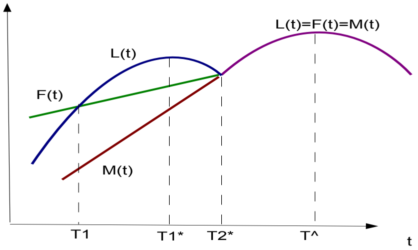
Leader and follower in a duopoly model

- In a duopoly, once one of the firm has entered, the choice of the other firm is a simple decision problem: if $t < T_2^*$, wait and enter at T^* and if $t \geq T_2^*$, enter immediately
- We compute the value of the leader entering at t: $L(t) = V(t, T_2^*)$ if $t < T_2^*$ and $L(t) = V(t, t)$ if $t \geq T_2^*$
- Value of the follower if the leader enters at t: $F(t) = V(T_2^*, t)$ if $t < T_2^*$ and $F(t) = V(t, t)$ if $t \geq T_2^*$
- Value if both enter at t: $M(t) = V(t, t)$

Case A: $L(T_1^*) > F(\hat{T})$



Case B: $L(T_1^*) < F(\hat{T})$



Equilibrium

Proposition

- *If $L(T_1^*) > M(\hat{T})$, there exists a unique equilibrium distribution of outcomes. With probability $\frac{1}{2}$, firm 1 adopts at T_1 and firm 2 at T_2^*. With probability $\frac{1}{2}$, the roles are reversed. Thus the equilibrium exhibits diffusion and with probability one, adoption times are T_1 and T_2^*.*
- *If $L(T_1^*) \leq M(\hat{T})$, there are two classes of equilibria. The first class is the (T_1, T_2^*) diffusion equilibrium. The second class is a continuum of joint adoption outcomes induced by any date of adoption $t \in (S, \hat{T})$ where $M(S) = L(T_1^*)$.*

Intuition for the preemption equilibrium

- Consider a discrete time adopt/non adopt game

	A	N
A	$M(t), M(t)$	$L(t), F(t)$
N	$F(t), L(t)$	$W(t+\Delta t), W(t+\Delta t)$

Intuition for preemption

- If $F(t) > L(t)$ this is a *waiting game*: choosing N dominates choosing A and both fins wait
- If $F(t) < L(t)$, as $M(t) < F(t) < L(t)$, there is no pure strategy equilibrium where both firms enter. Instead in a mixed strategy where firms adopt with probability $\pi(t)$,

$$\begin{aligned} W(t) &= \pi(t)F(t) + (1 - \pi(t))W(t + \Delta t) \\ &= \pi(t)M(t) + (1 - \pi(t))L(t). \end{aligned}$$

- As $\Delta \to 0$, $W(t + \Delta t) \to W(t)$.

Intuition for preemption

- We compute
$$\pi(t) = \frac{L(t) - F(t)}{L(t) - M(t)}.$$
- At T_1, the probability of exit goes to zero.
- However, as time intervals become smaller, the probability that one firm enters right at T_1 goes to 1
- However the probability that both firms enter simultaneously goes to zero.

Properties of equilibrium

- The payoffs of the two firms are equal
- The joint adoption equilibrium (when it exists) Pareto dominates the pre-commitment equilibrium
- The pre commitment equilibrium Pareto dominates the diffusion equilibrium
- In the diffusion equilibrium, both firms adopt too early.

Equilibrium with more than two firms

- With more than two firms, rent equalization does not necessarily obtain
- Suppose that we are in case A but after one firm enters, we turn to case B for the two other firms, so that there exists $\overline{T}$ such that $L(\overline{T}) = M(\hat{T})$.
- We use the fact that there are two possible equilibria after one firm enters (one equilibrium where the two other firms enter immediately, one where they wait until both adopt at $\hat{T}$).
- Construct an equilibrium where firm 3 adopts at $\overline{T}$ and firms $1, 2$ at $\hat{T}$. Firm 3 has a higher payoff than $1, 2$
- This is an equilibrium because if firm 1 or 2 preempts at $\overline{T} - \epsilon$, the two other firms adopt immediately, resulting in a payoff $M(\overline{T}) < M(\hat{T})$

A model of exit

- The previous models looked at the decision to enter
- Fudenberg and Tirole (1986) focus on the decision to exit
- In a model without uncertainty, firms never exit: they either choose not to enter or to enter
- In the model of FT, firms gradually learn about the cost of their competitors
- Firms choose to exit if they realize that their competitor has a low cost
- The model is a model of a *war of attrition* as analyzed by theoretical biologists.

The model

- Two firms on a market
- Instantaneous duopoly and monopoly profits are given by $D_i(t)$ and $M_i(t)$
- For most of the analysis, we assume that the industry is *expanding* $D_i' > 0, M_i' > 0$.
- Firms have opportunity costs θ_i if they exit the market.
- The opportunity cost is private information. The two opportunity costs are distributed independently according to a distribution $G(\theta_j)$ over $[\underline{\theta}, \overline{\theta}]$

Profits of the firms

- If firm 1 exits at $t_1 < t_2$, the profit is

$$\int_0^{t_1} D_1(t)e^{-rt}dt + e^{-rt_1}\theta_1.$$

- If firm 2 exits at $t_2 < t_1$, the profit is

$$\int_0^{t_2} D_1(t)e^{-rt}dt + \int_{t_2}^{\infty} M_1(t)e^{-rt}dt.$$

- The discounted expected values of monopoly and duopoly from time t on are

$$V_i^m(t) = \int_t^{|\infty} M_i(s)e^{-r(s-t)}ds, V_i^d(t) = \int_t^{\infty} D_i(s)e^{-r(s-t)}ds.$$

- Finally let $m_i = V_i^m(0), d_i = V_i^d(0)$.

Assumptions

- $D_i(t)$ and $M_i(t)$ are increasing converging asymptotically to $\overline{D_i}$ and $\overline{M_i}$
- $g_i(\cdot)$ is continuous, bounded away from zero.
- $\underline{\theta} < d_i$: it might be profitable for both firms to stay (different from the war of attrition)
- $m_i < \overline{\theta}$

Strategies

- A strategy specifies, for each value $\theta_i \in [\underline{\theta}, \overline{\theta}]$, the time $T_i(\theta_i)$ at which the firm chooses to exit if the other firm has not exited yet (similar to a war of attrition)
- The expected payoff of a firm of type θ_i who exits at t_i when the other firm chooses a strategy $T_j(\theta_j)$ is given by

$$\begin{aligned} V_i(t_i, Tj, \theta_i) &= \Pr[T_j(\theta_j) \geq t_i][\int_0^{t_i} D_i(t)e^{-rt}dt + \frac{e^{-rt_i}}{r}\theta_i] \\ &+ \int_{\theta_j | T_j(\theta_j) < t_i} [\int_0^{T_j(\theta_j)} D_i(t)e^{-rt}dt \\ &+ e^{-rT_j(\theta_j)} V_i^m(T_j(\theta_j))]g(\theta_j)d\theta_j. \end{aligned}$$

Equilibrium

- An equilibrium is a pair of strategies $(T_i(\theta_i), T_j(\theta_j))$ such that
$$V_i(\theta_i, T_i(\theta_i), T_j(\theta_j)) \geq V_i(\theta_i, t_i, T_j(\theta_j)) \forall t_i, \theta_i.$$

Characterization of equilibrium

- $T_i(\theta_i)$ is continuous, strictly decreasing with a differentiable inverse function $\Phi_i(t)$, which indicates the type of firm which exits at date t.
- Given that the other firm adopts the strategy $\Phi_i(t)$, what is the marginal cost/benefit for firm j to wait between t and $t + dt$?
- Marginal cost: $\Phi_j(t) - D_j(t)$
- Marginal benefit : $\frac{g(\Phi_i(t))\Phi_i'(t)}{G(\Phi_i(t))}[V_j^m(t) - \frac{\Phi_j(t)}{r}]$.

Characterization of equilibrium

- We obtain the differential equation

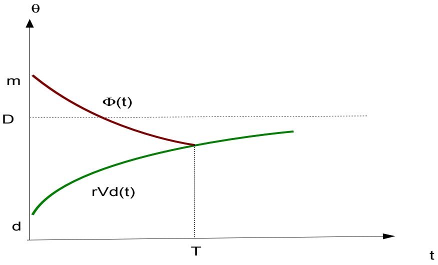
$$\Phi_i'(t) = -\frac{G(\Phi_i(t))}{g(\Phi_i(t))} \frac{\Phi_j(t) - D_j(t)}{V_j^m(t) - \frac{\Phi_j(t)}{r}}.$$

- and the boundary condition $\Phi_i(0) = m_i$
- This is the differential equation of a war of attrition which typically has many solutions (because the differential equation is not Lipschitz continuous at 0, as $V_j^m(t) - \frac{\Phi_j(t)}{r}$ goes to 0)
- The trick is to use the fact that with some probability the game will end with no exit (because $\underline{\theta} < d_i$) to compute the finite time at which firms will stop exiting, and use backward induction.

The two types of equilibria

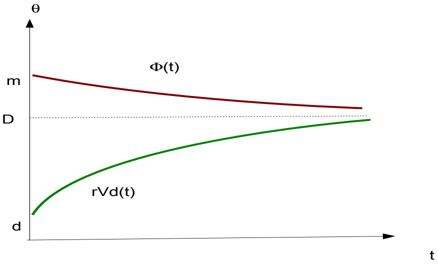
- First type: *Finite selection* (If $D_i(t)$ increases sufficiently fast): There is a finite date T at which firms stop exiting if they stay in the market until T
- Second type: *Perpetual selection*: At any date t, firms exit with positive probability. The probability of exiting goes to zero only when t goes to infinity.

Finite selection



Finite selection at T

Perpetual selection



Perpetual selection

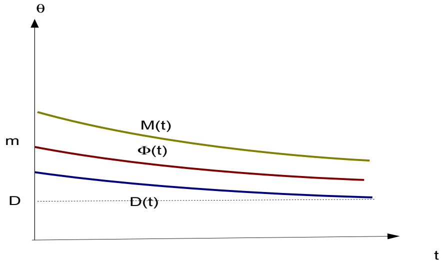
Properties of equilibrium

- Consider a change in the distribution from g to G' where $\frac{g}{G} < \frac{g'}{G'}$. The distribution G' puts more weight on high costs.
- The finite selection time is lower under G than G'. As costs increase, selection becomes slower.
- Suppose that duopoly and monopoly profits increase, $D' > D, M' > M$.
- The finite selection time is higher under D', M' than under D, M. As profits increase, selection becomes slower.

Declining industries

- The analysis can be repeated for D_i, M_i decreasing over time.
- In that case, there is never finite selection: selection is perpetual.

Declining industries



Declining industry