Dynamic oligopoly

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- Three papers by Eric Maskin and Jean Tirole (Econometrica and European Economic Review, 1987-1988)
- The papers discuss a general model of competition between firms based on an alternating move infinite horizon game.
- The authors discuss the Markov Perfect Equilibria of the game, and show that a number of observed phenomena (price cycles, kinked demand curves) can be explained through this unified model.
- The basic idea is short term commitment: each firm cannot change its action for a finite period of time.
  - Paper I: quantity competition with large fixed costs
  - paper II: price competition
  - Paper III: quantity competition without fixed costs

The Model

- Duopoly: Each firm chooses an action a^i
- Time is discrete $t = 1, 2, ...$ T is the time between two consecutive periods.
- Instantaneous profit π^i depends on the current actions but not on time:
$$\pi^i = \pi^i(a_t^1, a_t^2),$$
- Discount factor: $\delta = e^{-rT}$
- Firm i s intertemporal profit
$$\Pi^i = \sum_{t=0}^{\infty} \delta^t \pi^i(a_t^1, a_t^2)$$

Asynchronous choice

- Firm 1 chooses actions in odd periods $t = 1, 3, 5, ...$ and is committed to the action for two periods, $a^1_{2k+2} = a^1_{2k+1}$
- Firm 2 moves in even periods $t = 2, 4, 6, ..$ and $a^2_{2k+1} = a^2_{2k}$.
- Focus attention on Markov strategies, where players chose an optimal reaction function $R^1 : A \to A$, $R^2 : A \to A$
- Markov strategies (R^1, R^2) form a *Markov Perfect Equilibrium* of the game if and only if, at any period t, given the current state (a^1_t, a^2_t)
  - $R^1(a^2)$ maximizes the expected inter temporal profit of firm 1 from period t on
  - $R^2(a^1)$ maximizes the expected inter temporal profit of firm 2 from period t on

Dynamic programming

- Given an equilibrium pair of Markov strategies (R^1, R^2), define the Bellman equations, $V^1(a^2)$ – the value to firm 1 given that the last period 2 played a^2 and $W^1(a^1)$ – the value to firm 1 given that the last period 1 played a^1.
- $V^2(a^1)$ and $W^2(a^2)$ are defined accordingly.

$$
\begin{aligned}
V^1(a^2) &= \pi^1(R^1(a^2), a^2) + \delta W^1(R^1(a^2)), \\
W^1(a^1) &= \pi^1(a^1, R^2(a^1)) + \delta V^1(R^2(a^1)).
\end{aligned}
$$

Cournot competition

- Suppose that firms choose quantities (q^1, q^2) and face a fixed cost F. Let $f = F(1-\delta)$ the per period value of the fixed cost.
- Demand and costs are linear so that, if a firm produces, it obtains a profit

$$\pi^1(q^1, q^2) = q^1(1 - q^1 - q^2) - cq^1 - f.$$

- Fixed costs are so high that it is not efficient for two firms to coexist on the market: $2f > \pi^m > f$. where monopoly profit is $\pi^m = \frac{1-c}{4}$.

The static Cournot game

- In the static Cournot game there are three equilibria:
  - Two asymmetric equilibria where a single firm produces : $(q^m, 0), (0, q^m)$
  - A mixed strategy equilibrium in which each firm sets $q = \sqrt{f}$ with probability $\alpha = \frac{1-c}{\sqrt{f}} - 2$ and produces zero with probability $1 - \alpha$
- None of these models captures the idea that markets are *contestable*: the threat of entry drives the profit of the incumbent to zero!

Characterization of the MPE

Lemma

Equilibrium dynamic functions R^i are nonincreasing. If $q > q'$, then $R^i(q) \leq R^i(q')$.

Lemma

In any MPE, if 0 is a realization of $R^1(q)$ and $q > q'$ is a realization of $R^2(q')$ for some q', then $R^1(q) = 0$.

Lemma

In a symmetric MPE, if r is a positive realization of $R(q)$, $r > q$.

Characterization of the MPE

Lemma

In a symmetric MPE there exists $\overline{q}$ such that for all $q > \overline{q}$, $R(q) = 0$ and for all $q < \overline{q}$, there exists a positive realization of $R(q)$.

Lemma

In a symmetric MPE, for all q and all positive realizations r of $R(q)$, $R(r) = 0$.

Characterization of the MPE

$$\pi(q,q) + \frac{\delta}{1-\delta}\pi(q,0) = 0 \quad (1)$$

$$\arg\max_{\tilde{q}} \pi(\tilde{q},q) + \delta\pi(\tilde{q},0) = T(q), \quad (2)$$

$$\pi(q,q) + \delta\pi(q,0) + \frac{\delta^2}{1-\delta}(\pi^m - f) = 0 \quad (3)$$

$$\pi(T(q),q) + \delta\pi(T(q),0) + \frac{\delta^2}{1-\delta} = 0 \quad (4)$$

- Let q^*, q^{**} and q^{***} be the largest roots of equations (1), (3), (4).
- Let $\underline{q}$ be the solution to $T(\underline{q}) = q^{**}$.

The main proposition

Proposition

There exist numbers $\delta_1, \delta_2 \in (0,1)$ such that, if δ is the firms' discount factor, the unique symmetric MPE of the game is given by

- *If $\delta_1 \leq \delta < 1$, $R(q) = 0$ if $q \geq q^*$, $R(q) = q^*$ if $q < q^*$*
- *If $\delta_2 \leq \delta < \delta_1$, $R(q) = 0$ if $q \geq q^{**}$, $R(q) = q^{**}$ if $\underline{q} \leq q < q^{**}$,*
- *If $0 < \delta < \delta_2$, $R(q) = 0$ if $q \geq q^{***}$, $R(q) = T(q)$ if $q < q^{***}$.*

Analysis

- There is a deterrence level $\overline{q}$ such that if the firm is operating at or above this level, the other firm drops from the market.
- In at most three periods, one of the firms will drop out of the market.
- This deterrence level (denoted q^*, q^{**} and q^{***}) increases in the discount factor δ and decreases with the fixed cost f.
- When $\delta \geq \delta_1$, the deterrence level is above the monopoly profit q^m. In that case, the optimal strategy is to sell exactly the "limit quantity" $\overline{q}$.

Analysis

- As δ goes to 1, $\pi(\overline{q}, 0)$ tends to zero: this is like the Baumol, Panzar and Willig (1982) "contestable markets"
- If $\delta < \delta_1$, the deterrence quantity is lower than the monopoly quantity. In that case, either the firm chooses the monopoly quantity q^m and deters entry, or it selects the quantity $T(q)$ which maximizes the two period profit.

Asymmetric MPE

Proposition

There exist $\underline{\delta} \in (0,1)$ *and* $\underline{f} \leq \pi^m$ *such that, if* $\underline{\delta} \leq \delta < 1$ *and* $\underline{f} \leq f < \pi^m$*, there are exactly two asymmetric equilibria* $(R^1, R^2) = (T, 0)$ *and* $(R^1, R^2) = (0, T)$*.*

Cournot competition with no fixed costs

- Suppose again that the profit functions are quadratic

$$\pi^i = q_i(d - q_i - q_j),$$

- The static Cournot equilibrium is given by $q_1^s = q_2^s = \frac{d}{3}$ and $\pi_1^s = \pi_2^s = \frac{d^2}{9}$.

Characterizing equilibrium with differential functions

- Using the definitions of $V^1(q_2)$ and $W_1(q^1)$, we compute

$$\frac{\partial R_2}{\partial q_1} = \frac{-\pi_1^1(q_1, R_1^{-1}(q_1)) - \delta\pi_1^1(q_1, R_2(q_1))}{\delta\pi_2^1(q_1, R_2(q_1)) + \delta^2\pi_2^1(R_1(R_2(q_1)), R_2(q_1))}$$

$$\frac{\partial R_1}{\partial q_2} = \frac{-\pi_2^2(R_2^{-1}(q_2), q_2) - \delta\pi_2^2(R_1(q_2), q_2,)}{\delta\pi_1^2(R_1(q_2), q_2) + \delta^2\pi_1^2(R_1(q_2), R_2(R_1(q_2)))}$$

Looking for the equilibrium

- We look for a linear equilibrium:

$$R_i(q_j) = a_i - b_i q_j.$$

- If the equilibrium is symmetric, $a_i = a_j = a$, $b_i = b_j = b$
- We can easily invert the function $R(\cdot)$ and obtain two equations determining a and b:

$$\delta^2 b^4 + 2\delta b^2 - 2(1+\delta)b + 1 = 0, \quad (5)$$

$$a = \frac{1+b}{3-\delta b} d \quad (6)$$

The equilibrium

Proposition

For any discount factor δ, there exist a unique linear MPE where a and b are given by equations (5) and (6). This MPE is dynamically stable: i.e. for any history of the game the production levels converge to steady state outputs (q^e, q^e). When δ goes to zero, q^e converges to the static Cournot output $\frac{d}{3}$.

Lemma

The functions $a(\delta)$ and $b(\delta)$ are differentiable and $\frac{\partial b}{\partial \delta} < 0$, $\frac{\partial \delta b}{\partial \delta} > 0$ and $\frac{\partial a}{\partial \delta} < 0$.

Comparative statics

- As δ grows, a and b go down and q^e increases. The dynamic model of Cournot competition is more competitive than the static model.

Finite horizon vs infinite horizon

- It is impossible to show that the symmetric linear equilibrium is the unique equilibrium of the game
- If we consider the limit of finite horizon games, we can show that the linear MPE is the unique limit of any sequence of equilibria of the finite game.

Adjustment costs

- Suppose that adjustment costs are incurred each time a firm changes its capacity.
- Then a Markov strategy depends $q_{i,t}$ depends both on $q_{j,t-1}$ and on $q_{i,t-2}$.
- A linear MPE is then characterized by three parameters, a,b and β such that

$$R_i(q_{j,t-1}, q_{i,t-2}) = a - bq_{j,t-1} + \beta q_{i,t-2}.$$

- Adjustment costs make q^e go down: the market is less competitive with adjustment costs.

Edgeworth cycles

- In an Edgeworth cycle, firms undercut each other to increase their market share (price war phase)
- The war becomes too costly, and one of the firms increases price
- The other firms follow suit (relenting phase)
- Then price cutting starts again.
- The idea is due to Edgeworth (1925)

Kinked demand curve

- There is a focal point for prices
- Each firm believes that, if it undercuts, the other firms will immediately follow suit
- Each firm believes that, if it raises its price, the other firms will not follow.
- So firms have an incentive to set the price at the focal point.
- The idea is due to Sweezy (1939).

Price competition

- Firms choose prices p_t^1 and p_t^2 and receive instantaneous profit $\pi^i(p_t^1, p_t^2)$.
- The price space is discrete, goods are perfect substitutes, firms share the market equally if they set the same price.
- Firms have the same unit cost c.
- The demand function is given by $D(p)$.
- Let $\Pi(p) = (p - c)D(p)$

Profits

- Firm *i*s instantaneous profit is

$$\begin{aligned} \pi^i(p_t^1, p_t^2) &= \Pi(p_t^i) \text{ if } p_t^i, p_t^j, \\ &= \frac{\Pi(p_t)}{2} \text{ if } p_t^i = p_t^j = p_t, \\ &= 0 \text{ if } p_t^i > p_t^j. \end{aligned}$$

- The inter temporal profit is

$$\sum_{s=0}^{\infty} \delta^s \pi^i(p_{t+s}^1, p_{t+s}^2).$$

- In a MPE, the value functions are given by

$$\begin{aligned} V^1(\hat{p}) &= \max_p [\pi^1(p, \hat{p}) + \delta W^1(p)], \\ W^1(\hat{p}) &= E_p[\pi^1(\hat{p}, p) + \delta V^1(p)] \end{aligned}$$

Example

- In this example, $D = 1 - p$, $c = 0$.
- Firms can charge any of seven prices $p(i) = \frac{i}{6}$, $i = 0, 1, 2, ...6$
- The corresponding profits are proportional to $0, 5, 8, 9, 8, 5, 0$
- The monopoly price is $p^m = p(3) = \frac{1}{2}$.

Example: Kinked demand curve

- Let $\beta(\delta) = \frac{5+\delta}{5\delta+9\delta^2}$.

$\Pi(p)$	p	$R(p)$
0	$p(6)$	$p(3)$
5	$p(5)$	$p(3)$
8	$p(4)$	$p(3)$
9	$p(3)$	$p(3)$
8	$p(2)$	$p(1)$
5	$p(1)$	$p(1)$ prob $\beta(\delta)$, $p(3)$ other
0	$p(0)$	$p(3)$

Example: Kinked demand curve

- *For δ sufficiently close to one, this symmetric strategy profile forms a MPE*
- No incentive to undercut at $p(3)$.
  - If choose $p(3)$, you get $V(3) = \frac{9}{2}\frac{1}{1-\delta}$
  - If undercut at $p(2)$, you get $8 + \delta * 0 + \delta^2 V(1) = 8 + \delta^2 \frac{9}{2}(\delta + \delta^2 + ...)$.
- For δ sufficiently close to 1, you prefer to stay at $p(3)$
- At $p(2)$: prefer to start the price war rather than move to $p(3)$
  - By choosing $p(2)$, get $5 + \delta W(1)$. Because at $p(1)$, firms are indifferent between switching to $p(3)$ or staying at $p(1)$, $V(1) = \frac{9}{2}\frac{\delta}{1-\delta} = \frac{5}{2} + \delta W(1)$. So switching to $p(1)$ gives $\frac{5}{2} + \frac{\delta}{1-\delta}\frac{9}{2}$.
  - Relenting to $p(3)$ gives $\frac{9}{2}\frac{\delta}{1-\delta}$.

Kinked demand curve: Interpretation

- In this example, ultimately the price reaches the monopoly price $p(3)$
- If a firm raises its price, the other firm does not follow suit, and the firm loses its customers.
- If a firm undercuts, the other firm immediately follows suit, resulting in a price war.
- Scherer notes that "prices tend to be at least as rigid downward as they are upward in well disciplined oligopolies": this seems to contradict our findings.
- One possible explanation is that firms set prices below the profit maximizing level so that rigidities appear both upward and downward.

Example: Edgeworth cycles

- Let $\alpha(\delta) = \frac{(3\delta^2-1)(1+\delta^2+\delta^4)}{8+7\delta^2+2\delta^4+3\delta^6}$.

$\Pi(p)$	p	$R(p)$
0	$p(6)$	$p(4)$
5	$p(5)$	$p(4)$
8	$p(4)$	$p(3)$
9	$p(3)$	$p(2)$
8	$p(2)$	$p(1)$
5	$p(1)$	$p(0)$
0	$p(0)$	$p(0)$ proba $\alpha(\delta)$, $p(5)$ other.

Example: Edgeworth cycles

- *For δ sufficiently close to one, this symmetric strategy profile forms a MPE*
- Firms undercut each other successively until they reach the competitive price $p(0)$
- At price $p(0)$, firms engage in a war of attrition. Each firm relents with positive probability and moves back to price $p(5)$
- Firm's incentives to undercut are clear.
- At the competitive price, firms wait for their competitor to relent first: this war of attrition explains why firms play a mixed strategy.
- Notice that Edgeworth cycles are obtained *without capacity constraints* whereas in Edgeworth's original contribution, capacity constraints led to price cycles.

General results: Lemmata

Lemma

The valuation function V^i is nondecreasing.

- A price is focal if the firms continue to charge it forever (absorbing state of the Markov chain)

$$p^f = R^1(p^f) = R^2(p^f).$$

Lemma

If p^f is a focal price, $\pi(p^f) > 0$.

General results: Lemma

- A price is semi focal if it is in the support of both $R^1(p^f)$ and $R^2(p^f)$.

Lemma

If $\Pi(p^f) > 0$, a firm never reacts to a price p above a focal or semi focal price p^f by undercutting to a price $\hat{p} < p^f$ or by raising its price. Thus the support of $R^i(p)$ lies in the interval $[p^f, p]$.

Markov chains and ergodic sets

- Any (possibly) mixed Markov strategies R^1, R^2 induce a Markov chain over the set of prices.
- An ergodic set is a maximal set of states which communicate (there is a positive probability of transition between any two states)
- If an ergodic set is a singleton (focal price), this is equivalent to a *kinked demand curve*
- If an ergodic set has multiple states, we call it an *Edegworth cycle*

General results

Proposition

For a given price grid, an MPE cannot have two focal ergodic classes if the discount factor is close enough to 1.

Proposition

An MPE cannot possess both a focal and an Edgeworth ergodic class.

- These propositions show that an MPE falls into one of two categories (KDC or EC).
- It does not rule out the fact that the same MPE may give rise to two different EC according to the initial conditions.

KDC: General results

- Let k be the step of he price grid. Define two prices $x < p^m < y$ such that

$$\begin{aligned} \Pi(x) > & \tfrac{4}{7}\Pi(p^m) & \geq \Pi(x-k) \\ \Pi(y) > & \tfrac{2}{3}\Pi(p^m) & \geq \Pi(y+k) \end{aligned}$$

Proposition

If p^f is a focal price of some MPE, then for a high discount factor, $p^f \leq y$ and for a sufficiently fine grid $p^f \geq x$

Proposition

For a sufficiently fine grid and a price p belonging to the grid in $[x, y]$, p is the focal price of some MPE for a discount factor close enough to one.

SMKE

- If the focal price is the monopoly price p^m, there is a simple equilibrium (SMKE)
  - Each firm cuts its price to p^m when the market price is above p^m
  - Each firm cuts its price to a relenting price $\underline{p}$ when the price is between $\underline{p}$ and p^m
  - Each firm raises its price to p^m when the price is below $\underline{p}$.
- The relenting price must satisfy

$$4\Pi(\underline{p}) \geq \Pi(p^m) > 4\Pi(\underline{p} - k).$$

Monopoly profits and SMKE

Proposition

For a sufficiently fine grid, sufficiently high discount factor, the unique MPE for which aggregate profit per period $(1-\delta)\frac{(V^i(p)+W^j(p))}{2}$ *is within* ϵ *of* $\Pi(p^m)$ *is the SMKE.*

- An MPE is *renegotiation proof* if at any price p there is no MPE which Pareto dominates it.
- (A similar concept exists for repeated games and implicit collusion)

Proposition

For a sufficiently fine grid, sufficiently high discount factor, the unique renegotiation proof MPE is the SMKE.

Edgeworth cycles

Proposition

Assume that the profit function $\Pi(p)$ is strictly concave. For a fine grid and discount factor near 1, there exists an Edgeworth cycle.

- The prototypical equilibrium is characterized by two prices $\underline{p}$ and $\overline{p}$ such that

$$\begin{aligned} R(p) &= \overline{p} \text{ for } p > \overline{p}, \\ &= p - k \text{ for } \overline{p} \geq p > \underline{p} \\ &= c \text{ for } \underline{p} \geq p > c, \\ &= c \text{ with prob } \mu(\delta), \overline{p} + k \text{ other} \end{aligned}$$

Profit bounds and Edgeworth cycles

Proposition

For a discount factor near one and a sufficiently fine grid, at least one firm earns average profit no less than a quarter of monopoly profit in an MPE. In a symmetric MPE, this must be true for both firms.

Price vs quantity competition

- Quantity competition: single MPE
- Price competition: multiple MPE
- The difference is due to different signs of the cross partial Π^i_{12} (negative in Cournot, nonmonotonic in Bertrand)
- In quantity competition, an increase in δ results in an increase in quantities, making market more competitive.
- In price competition, an increase in δ makes it more likely to sustain high prices.
- If firms could choose the time period between moves, they would choose long periods in the quantity model but short periods in the price model.

Endogenous timing

- Here we assume that firms have to choose an action every other period.
- Alternatively, we could let firms decide whether to not to change price/quantities very period, and be committed to the same action for two periods.
- In this alternative model, timing of actions is endogenous.
- We distinguish in the state space between states where the other firm is committed to a price or not.
- With this new formulation, the MPE remain robust: in the quantity game, firms move to alternating moves in finite time
- For price games, firms will also move to alternating moves because they earn zero profit in the simultaneous game.

Markov behavior and tacit collusion

- In the asynchronous dynamic oligopoly model, firms are able to sustain high profits close to monopoly
- this result is reminiscent of implicit collusion
- However, in the asynchronous model, firms play Markov strategies (which do not depend on the entire history)

Excess capacity and market sharing

- In the KDC example, firms must have enough capacity for the price war. Otherwise they cannot sustain the monopoly price.
- If firms choose capacities, they have an incentive to build excess capacity.
- A firm may voluntarily choose to reduce its market share (not supply all demand) in order to limit the aggressive behavior of its competitor in the future.
- To avoid triggering a price war, firms may choose to limit their sales.