

Economics of Information

Problem Set 4

1. A seller owns one unit of a good which she values at c . A buyer may buy the unit from the seller. The seller's valuation is equal to \underline{c} or \bar{c} with equal probability and is private information to the seller. The buyer's valuation for the good is \bar{v} if $c = \bar{c}$ and \underline{v} if $c = \underline{c}$ where $\bar{v} > \bar{c}$ and $\underline{v} > \underline{c}$. The buyer thus has no private information. Assume that $\frac{\underline{v} + \bar{v}}{2} < \bar{c}$ (which implies that $\underline{v} < \bar{c}$). Show that efficiency is inconsistent with the seller's and buyer's incentive compatibility and individual rationality. With the quality interpretation in mind, suppose that there is a continuum of buyers and sellers. Buyers are homogeneous and have the same valuation for the good (which is either equal to \underline{v} or \bar{v}). Each seller has probability $\frac{1}{2}$ of having a high quality item. Qualities are independent across sellers. Show that the inefficiency result carries over – this is Akerlof's lemons problem.
2. A firm's profit is $x = \theta + e$ where e is the manager's effort and θ a productivity parameter only known to the manager. θ takes value $\underline{\theta}$ with probability \underline{p} and $\bar{\theta}$ with probability \bar{p} . The manager's objective is $u_1 = t - g(e)$ and the shareholder's utility function is $u_1 = x - t - Kq$ where q is the probability of audit and K the cost of auditing. The shareholder offers a contract $\{x(\theta), t(\theta), q(\theta)$ where θ is the firm's announcement of its productivity parameter. If it announces θ , the firm is required to attain profit level $x(\theta)$. After production takes place, the shareholder audits with probability $q(\theta)$. The audit yields a signal $\tilde{\theta} \in \{\underline{\theta}, \bar{\theta}\}$. The probability that the signal is truthful is $r \in [\frac{1}{2}, 1]$. If $\tilde{\theta} = \theta$, the manager receives $t(\theta)$. If $\tilde{\theta} \neq \theta$, the manager, who is protected by limited liability, receives 0. Show that $q(\bar{\theta}) = 0$. Show that for K not too big, auditing always occurs for r close to 1 and $\theta = \underline{\theta}$. Show that when K varies there are three regimes (including one in which the first best effort is obtained). Indicate how $x(\underline{\theta})$ changes with r and K . Explain.