Economics of Information

Problem Set I

1. There are two states s_1 and s_2 and two messages m_1 and m_2. The prior probability distribution over states is $\pi = (0.7, 0.3)$. The posterior probabilities $\pi_{s|m}$ include $\pi_{1|1} = 0.9$ and $\pi_{2|2} = 0.8$. Calculate the Π, L and J matrices. For the prior probability distribution π above, and still assuming two possible messages, show (if it is possible to do so):

    (a) AN L matrix representing completely conclusive message service (leaving no posterior uncertainty whatsoever). If it exists is it unique or are there other such L matrices? Also, does any such matrix depend at all upon π?

    (b) An L matrix that is completely uninformative. Answer the same questions.

    (c) An L matrix that (if one message is received) will conclusively establish that one of the states will occur but (if the other message is received) will be completely uninformative. Same questions.

2. You are the receiving officer of a company that has received a large shipment of ordered goods. You must decide whether to accept (action $x = A$) or reject (action $x = R$) the shipment. Which you will want depends upon the unknown proportion p of defective goods in the shipment. Your loss function is $L(R, p) = 0$ for $p \geq 0.04$ and $L(R, p) = 100(0.4 - p)$ for $p < 0.04$ and $L(A, p) = 0$ for $p \leq 0.04$ and $L(A, p) = 200(p-0.04)$ for $p \geq 0.04$. Your prior probability distribution is defined over four discrete values of p:

fraction defective	0.02	0.04	0.06	0.08
prior probabaility	0.7	0.1	0.1	0.1

    (a) What is your best prior decision in the absence of sample information?

    (b) How much are you willing to pay for a sample of size 1?

3. An individual with initial wealth W faces state-contingent prices p_s for $s = 1, 2$. His utility function is $u(c) = \log c$.

    (a) If he believes that state 1 will occur with probability π and state 2 with probability $1 - \pi$ obtain an expression for his maximized expected utility $U(\pi)$ as a function of π. Depict this in a diagram for $0 \leq \pi \leq 1$.

(b) Is $U(\pi)$ convex?

(c) Suppose that the prior probability of state 1 is $\overline{\pi}$ and that he expects to receive ont of two messages that will change the probability of state 1 either to $\overline{\pi}+\theta$ or to $\overline{\pi}-\theta$. Explain why the probabaility of each message must be 0.5 and show that the gain in expected utility from having this information prior to trading is

$$\begin{aligned}
\Omega(\theta) &= \frac{1}{2}(\overline{\pi}+\theta)\log(\overline{\pi}+\theta)+\frac{1}{2}(\overline{\pi}-\theta)\log(\overline{\pi}-\theta) \\
&- \overline{\pi}\log\overline{\pi}+\frac{1}{2}(1-\overline{\pi}-\theta)\log(1-\overline{\pi}-\theta)+\frac{1}{2}(1-\overline{\pi}+\theta)\log(1-\overline{\pi}+\theta) \\
&- (1-\overline{\pi})\log(1-\overline{\pi})
\end{aligned}$$

(d) Hence show that the marginal gain in utility from a little bit of information is zero. Also show that for all $\theta > 0$ the marginal value $\Omega'(\theta)$ is positive.

4. Consider the following two-consumer, two-commodity general equilibrium model. The utility functions of the two consumers are

$$\begin{aligned}
u_1(x_{11},x_{21}) &= x_{11}+x_{21} \\
u_2(x_{12},x_{22}) &= \sqrt{x_{12}}+x_{22}
\end{aligned}$$

Consumer 1's endowment of the second good is ω_{21}. He has no endowment of the first good. Consumer 2 has no endowment of the second good and his endowment of the second good depends on which of three equally likely states occur. The respective levels in the three states are $\omega_{112}, \omega_{122}, \omega_{132}$.

(a) Determine the Arrow-Debreu equilibrium of this economy

(b) Suppose that before any trade takes place, consumers are told whether or not state 1 has occurred. After the revelation of this information contingent trades are permitted.

(c) Compare the ex ante utility of the agents when they receive the signal and when they do not receive the signal.